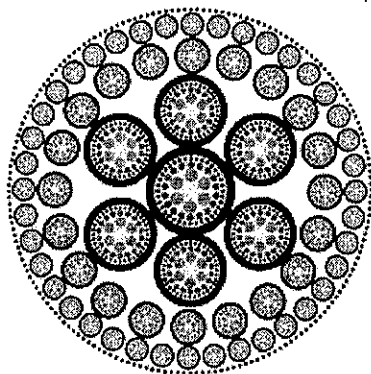


Geometry

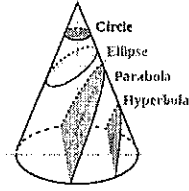
Unit 4-13 Circles

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Lesson 2: Segments within a Circle	pages 7-12	HW page 13
Lesson 3: Circle Segments and Similarity	page 14-15	HW page 16-17
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Lesson 5: Angles formed by Intersecting Chords	pages 25-26	HW pages 27-28
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Unit 13 Circles

Lesson 1 Equations of Circles

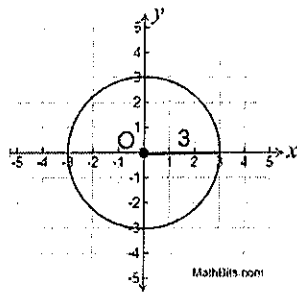


A circle is a conic section.
 It is a slice of a right cone parallel to the circular base of the cone.
 In this lesson, we will be examining this "circular slice"
 as to its properties and equations in relation to the coordinate plane.

Circle with Center at Origin (0,0)

$$x^2 + y^2 = r^2$$

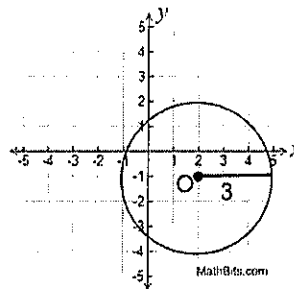
with the center at (0,0)
and the radius r



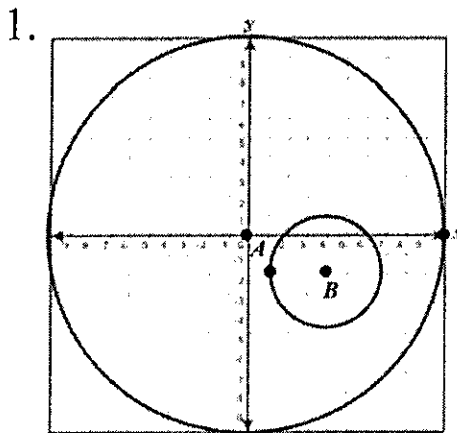
Circle with Center at Point (h, k) (Known as "center-radius form" or "standard form".)

$$(x - h)^2 + (y - k)^2 = r^2$$

with the center at (h, k)
and the radius r

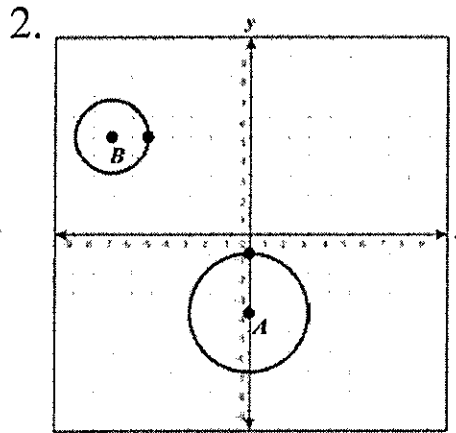


Examples (graph):



Equation of Circle A:

Equation of Circle B:



Equation of Circle A:

Equation of Circle B:

Examples (center/radius):

3. Write the equation of the circle with center $(2, -8)$ and radius 6.
4. Write the equation of the circle with center $(-5, 4)$ and radius 11.
5. Write the equation of the circle with center $(0, -7)$ and diameter 20.
6. Write the equation of the circle with center at the origin and diameter 16.
7. Given the equation of the circle $(x - 5)^2 + (y + 3)^2 = 196$,
what is the center and radius of the circle?
8. Given the equation of the circle $(x + 9)^2 + y^2 = 144$,
what is the center and diameter of the circle?

Examples (center/point):

9. Write the equation of the circle with center at origin passing through the point $(-4, 0)$.
10. Write the equation of the circle with center at origin passing through the point $(-5, 12)$.
11. Write the equation of the circle with center $(-4, 8)$ passing through the point $(-2, -1)$.
12. Write the equation of the circle with center $(-1, 2)$ passing through the point $(1, 2)$.

Examples (endpoints of diameter):

13. Write the equation of the circle whose diameter has endpoints (2, 0) and (2, -8).

14. Write the equation of the circle whose diameter has endpoints (-2, 3) and (6, 3).

15. Write the equation of the circle whose diameter has endpoints (4, -2) and (4, -4).

16. Write the equation of the circle whose diameter has endpoints (0, 2) and (-2, -2).

$$x^2 + y^2 + Cx + Dy + E = 0$$

General Form

Example 17:

Re-write the equation so that it is in general form

$$(x - 2)^2 + (y + 1)^2 = 9$$

Example 18: Rewrite the equation so that it is in center radius form.

$$x^2 + y^2 + 4x - 16y + 52 = 0$$

- Start by grouping the x -related terms together and the y -related terms together. Move any numerical constants (plain numbers) to the other side.
- Get ready to insert the needed values for creating perfect square trinomials. Remember to balance both sides of the equation.
- Find the missing value by taking half of the "middle term" (the linear coefficient) of the trinomial and squaring it. This value will always be positive as a result of the squaring process.
- Rewrite in factored form.

Identify the center and radius of each circle

Example 19: $x^2 + y^2 + 2x + 18y + 1 = 0$

Example 20: $x^2 - 14x + y^2 - 2y - 50 = 0$

Example 21: $x^2 + y^2 - 10x + 10y = -48$

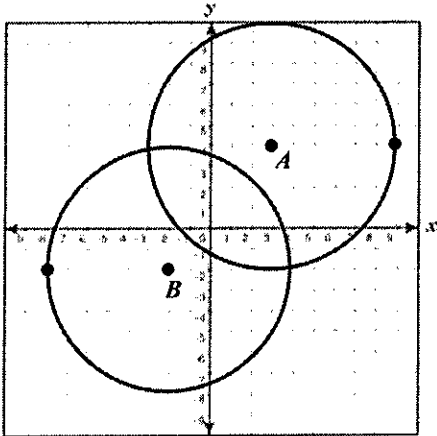
Unit 13 Lesson 1: Equations of Circles HW

Geo CC

1. Determine the center and radius of the given circles.

- a) $(x-7)^2 + (y+10)^2 = 81$ Center (____, ____)
 Radius = ____
- b) $100 = (x+3)^2 + y^2$ Center (____, ____)
 Radius = ____
- c) $(x-9)^2 + (y+2)^2 = 1$ Center (____, ____)
 Radius = ____

2. Determine the equation of the circle.



Equation of Circle A:

Equation of Circle B:

3. What is the equation of a circle whose center is 4 units above the origin in the coordinate plane and whose radius is 6?

- 1) $x^2 + (y-6)^2 = 16$
- 2) $(x-6)^2 + y^2 = 16$
- 3) $x^2 + (y-4)^2 = 36$
- 4) $(x-4)^2 + y^2 = 36$

4. The center of a circular sunflower with a diameter of 4 centimeters is $(-2, 1)$. Which equation represents the sunflower?

- 1) $(x-2)^2 + (y+1)^2 = 2$
- 2) $(x+2)^2 + (y-1)^2 = 4$
- 3) $(x-2)^2 + (y-1)^2 = 4$
- 4) $(x+2)^2 + (y-1)^2 = 2$

5. A circle whose center has coordinates $(-3, 4)$ passes through the origin. What is the equation of the circle?

- 1) $(x+3)^2 + (y-4)^2 = 5$
- 2) $(x+3)^2 + (y-4)^2 = 25$
- 3) $(x-3)^2 + (y+4)^2 = 5$
- 4) $(x-3)^2 + (y+4)^2 = 25$

6. Write an equation of a circle whose diameter has endpoints $(-2, -1)$ and $(2, 3)$.

7. Determine the center and radius of the given circles by completing the square.

a) $x^2 + 10x + y^2 - 16 = 0$

b) $x^2 + y^2 + 18x + 17 = 0$

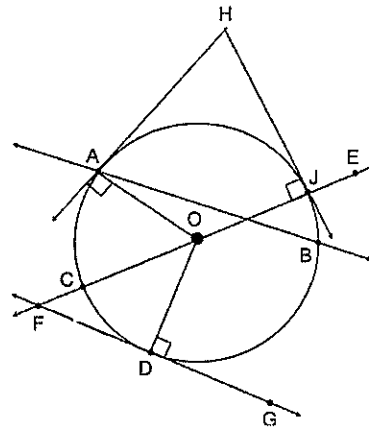
c) $x^2 + 2x - 18 = -y^2 + 8x$

Unit 13 Lesson 2

Segments within a circle

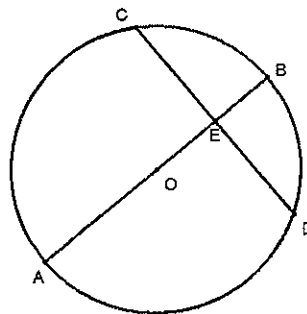
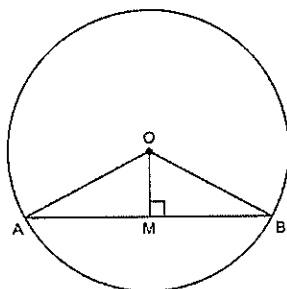
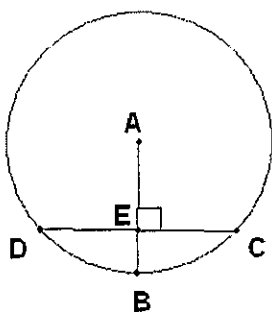
Given Circle O

1. *Radius*:
 - a line segment between the center and a point on the circle.
2. *Chord*:
 - a line segment in the interior of the circle with both endpoints on the circle.
3. *Diameter*:
 - a chord passing through the center of the circle.
 - A diameter is the longest possible chord.
4. *Secant*:
 - a line passing through two points on the circle. A secant forms a chord in the interior of the circle.
5. *Tangent*:
 - a line touching (intersecting) a circle at one point without entering the interior of the circle.
6. *Point of Tangency*:
 - the point at which a tangent touches (intersects) a circle.
7. *Arc*:
 - a part of a circle's circumference.
8. *Minor Arc*:
 - any arc whose length is less than the length of a semicircle (with a corresponding central angle less than 180°).
9. *Major Arc*:
 - any arc whose length is greater than the length of a semicircle (with a corresponding central angle greater than 180°). Major arc are usually named using three letters to distinguish them from minor arcs.
10. *Semicircle*:
 - an arc whose length is exactly half of the circumference (with a corresponding central angle equal to 180°).

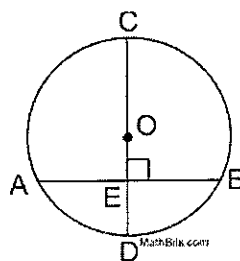


Radius Perpendicular to a chord

THEOREM: In a circle, a radius perpendicular to a chord bisects the chord.

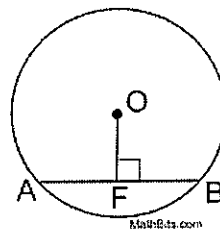


Example 1 Given circle O with diameter \overline{CD} perpendicular to chord \overline{AB} . If $AE = 8$ inches, find EB .



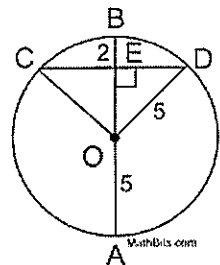
Example 2

Given circle O with perpendicular at F .
 $AF = 2x + 12$ and $FB = 4x - 40$.
 Find AB .



Example 3

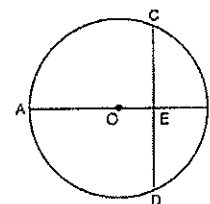
Given circle O with a radius of 5, and $BE = 2$.
 Diameter \overline{AB} is perpendicular to chord \overline{CD} .
 Find CD .



Example 4

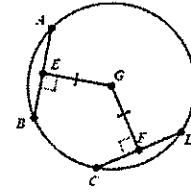
In the diagram below of circle O , diameter \overline{AOB} is perpendicular to chord \overline{CD} at point E .
 $OA = 6$, and $OE = 2$.

What is the length of \overline{CE} ?



Chords equidistant from the center of a circle

THEOREM: In a circle, or congruent circles, congruent chords are equidistant from the center.

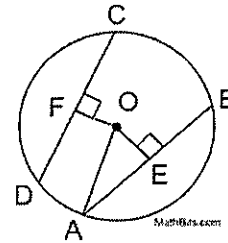


Example 5

If $EG = 4$, $GF = 4$, $AB = 5x + 3$, $CD = 2x + 9$, find x .

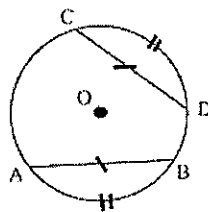
Example 6

Given circle O , $\overline{AB} \cong \overline{CD}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$.
If $OF = 16$, $DF = x + 10$, and $CD = 4x - 20$,
find OA .

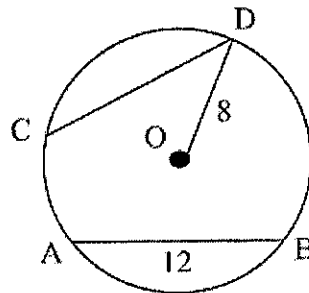


Congruent Chords

THEOREM: In a circle, or congruent circles, congruent chords have congruent arcs.



Example 7



Given: Circle

$$\widehat{AB} \cong \widehat{CD}$$

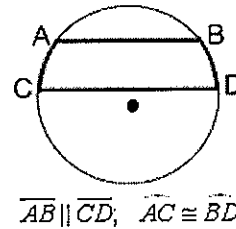
$$AB = 12$$

$$OD = 8$$

Find CD .

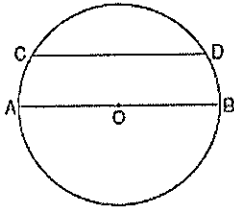
Parallel Chords

THEOREM: In a circle, parallel chords intercept congruent arcs



Example 8

In the diagram of circle O below, chord \overline{CD} is parallel to diameter \overline{AOB} and $m\widehat{AC} = 30$.



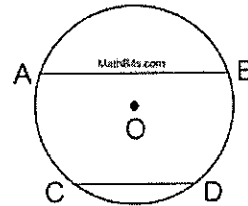
What is $m\widehat{CD}$?

Example 9

Given circle O with $\overline{AB} \parallel \overline{CD}$.

$m\widehat{AB} = 8x + 50$, $m\widehat{AC} = 5x + 30$, $m\widehat{BD} = 9x - 10$

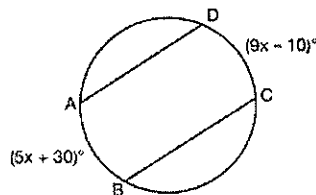
Find $m\widehat{CD}$.



Example 10

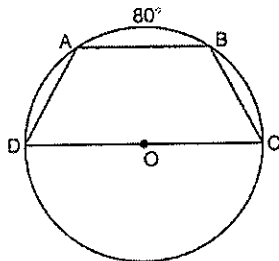
In the diagram of the circle below. $\overline{AD} \parallel \overline{BC}$, $m\widehat{AB} = (5x + 30)^\circ$, and $m\widehat{CD} = (9x - 10)^\circ$.

What is $m\widehat{AB}$?

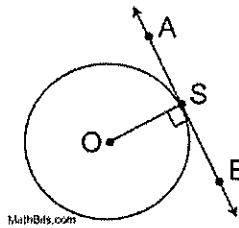


Example 11

In the diagram below, trapezoid $ABCD$, with bases \overline{AB} and \overline{DC} , is inscribed in circle O , with diameter \overline{DC} . If $m\widehat{AB} = 80$, find $m\widehat{BC}$.

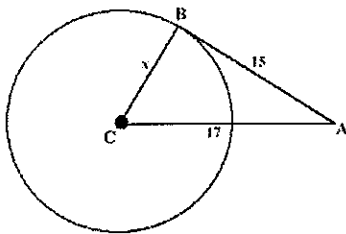


THEOREM: If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of tangency.



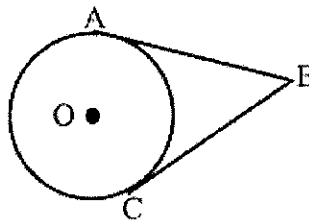
Example 12

Find the length of the radius.



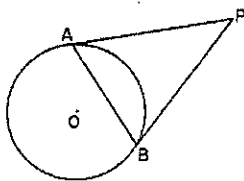
Tangents from a common exterior point

THEOREM: Tangent segments to a circle from the same external point are congruent. (You may see this theorem referred to as the "hat" theorem as the circle appears to be wearing a hat.)

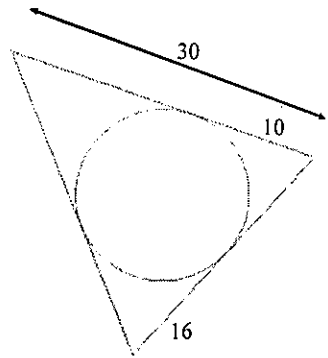


Example 13

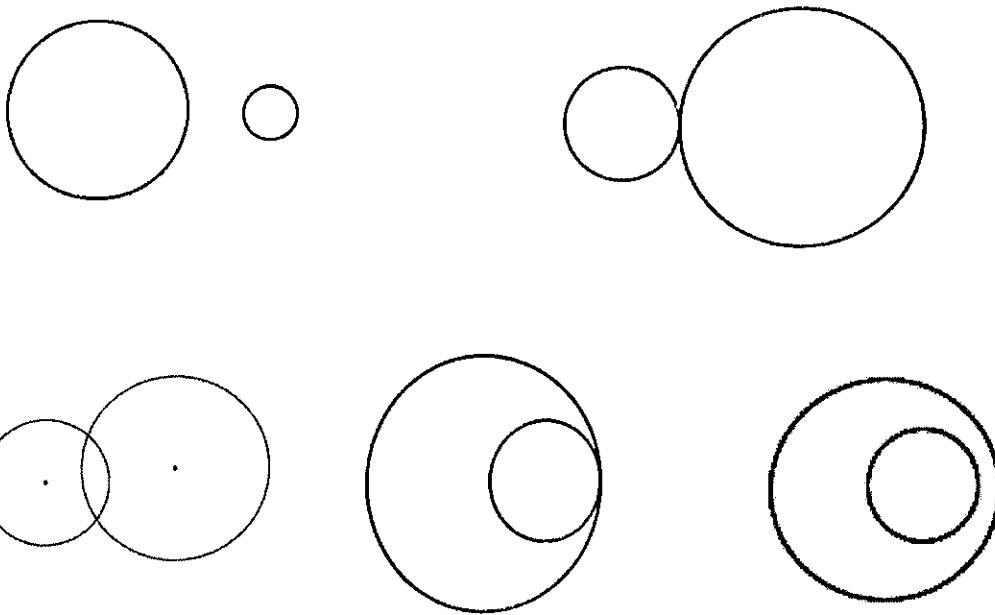
In the accompanying diagram, \overline{PA} and \overline{PB} are tangents drawn to circle O . If $m\angle PBA = 70$, find $m\angle P$.



Example 14 Find the perimeter of the triangle



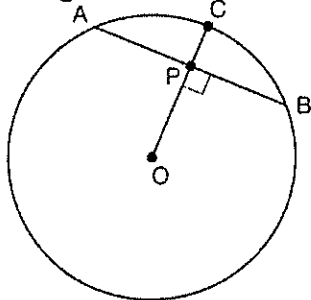
Common Lines of Tangency



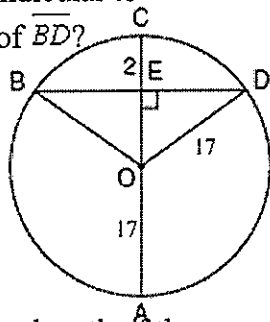
Unit 13 Lesson 2

Segments in a Circle HW

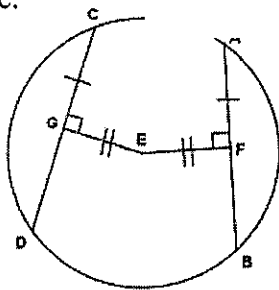
1. In the diagram below of circle O , radius \overline{OC} is 5 cm. Chord \overline{AB} is 8 cm and is perpendicular to \overline{OC} at point P . What is the length of \overline{OP} , in centimeters?



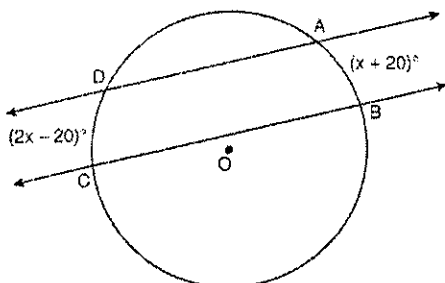
2. In the diagram below, circle O has a radius of 17, and $CE = 2$. Diameter \overline{AC} is perpendicular to chord \overline{BD} at E . What is the length of \overline{BD} ?



3. In circle E , $CD = 24$, $EF = 5$, find the length of the radius of the circle.

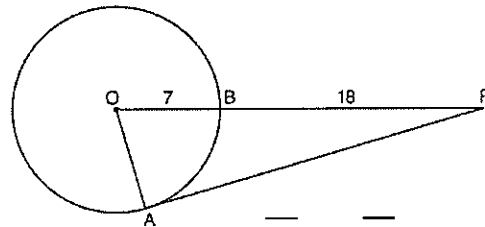


4. In the diagram below, two parallel lines intersect circle O at points A, B, C , and D , with $m\widehat{AB} = x + 20$ and $m\widehat{DC} = 2x - 20$. Find $m\widehat{AB}$.

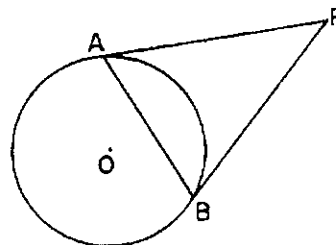


5. If $x^2 + 4x + y^2 - 6y - 12 = 0$ is the equation of a circle, what is the length of the radius?

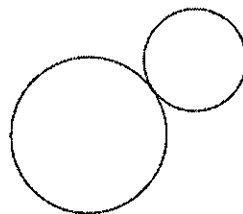
6. In the diagram below of $\triangle PAO$, \overline{AP} is tangent to circle O at point A , $OB = 7$, and $BP = 18$. What is the length of \overline{AP} ?



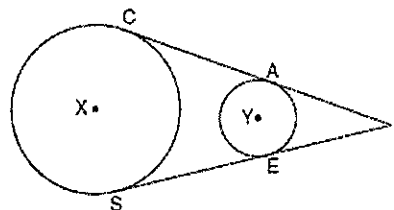
7. In the accompanying diagram, \overline{PA} and \overline{PB} are tangents drawn to circle O . If $m\angle PBA = 80^\circ$, find $m\angle P$.



8. How many common tangent lines can be drawn to the two externally tangent circles shown below?

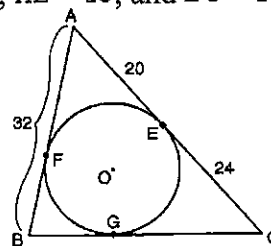


9. In the diagram below, circles X and Y have two tangents drawn to them from external point T . The points of tangency are C, A, S , and E . The ratio of TA to AC is $1:3$. If $TS = 24$, find the length of \overline{SE} .



(Not drawn to scale)

10. In the accompanying diagram, \overline{AFB} , \overline{AEC} , and \overline{BGC} are tangent to circle O at F, E , and G , respectively. If $AB = 32$, $AE = 20$, and $EC = 24$, find BC .

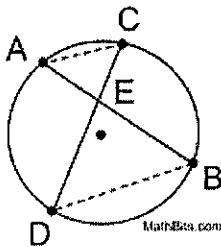


Unit 13 Lesson 3

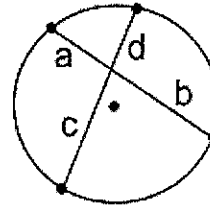
Circle Segments and Similarity

THEOREM: If two chords intersect in a circle, the product of the lengths of the segments of one chord equal the product of the segments of the other.

Justification:



- $\Delta EAC \sim \Delta EDB$
- $\frac{EC}{EB} = \frac{AE}{DE}$
- $AE \times EB = DE \times EC$

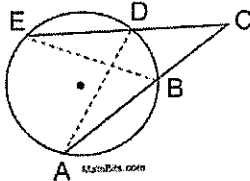


Intersecting Chords Formula:
 (segment piece) x (segment piece) =
 (segment piece) x (segment piece)

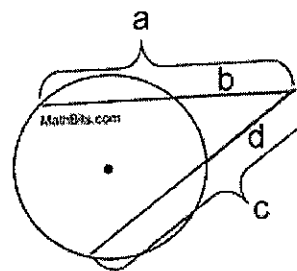
Formula: $a \cdot b = c \cdot d$

THEOREM: If two secant segments are drawn to a circle from the same external point, the product of the length of one secant segment and its external part is equal to the product of the length of the other secant segment and its external part.

Justification:



- $\Delta ECB \sim \Delta ADC$
- $\frac{EC}{AC} = \frac{BC}{DC}$
- $EC \cdot DC = AC \cdot BC$



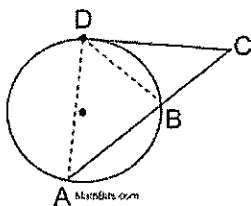
Secant-Secant Formula:
 (whole secant) x (external part) =
 (whole secant) x (external part)

Formula: $a \cdot b = c \cdot d$

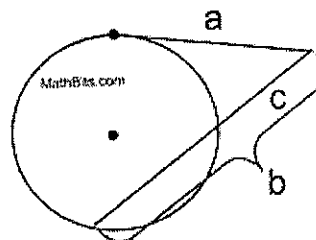
THEOREM:

the product of the length of the secant segment and its external part equals the square of the length of the tangent segment.

Justification:



- $\triangle CAD \sim \triangle CDB$
- $\frac{AC}{DC} = \frac{DC}{BC}$
- $AC \cdot BC = (DC)^2$

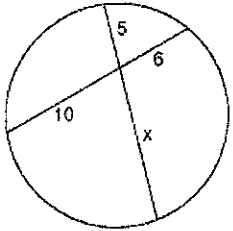


Secant-Tangent Formula:

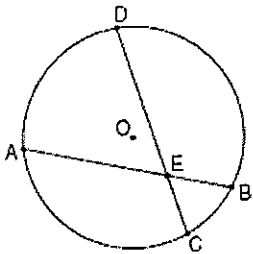
(whole secant) x (external part) = (tangent)²

Formula: $b \cdot c = a^2$

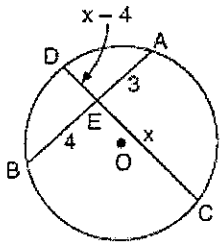
1. The accompanying diagram shows two intersecting paths within a circular garden. What is the length of the portion of the path marked x ?



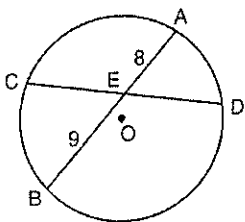
2. In the diagram of circle O below, chord \overline{AB} intersects chord \overline{CD} at E , $DE = 2x + 8$, $EC = 3$, $AE = 4x - 3$, and $EB = 4$. What is the value of x ?



3. In the accompanying diagram of circle O , chords \overline{AB} and \overline{CD} intersect at E . If $AE = 3$, $EB = 4$, $CE = x$, and $ED = x - 4$, what is the value of x ?

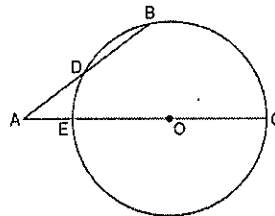


4. In the diagram below of circle O , chord \overline{AB} bisects chord \overline{CD} at E .
 If $AE = 8$ and $BE = 9$, find the length of \overline{CE} in simplest radical form.



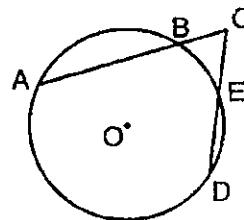
5. A toy truck is located within a circular play area. Alex and Dominic are sitting on opposite endpoints of a chord that contains the truck. Alex is 4 feet from the truck, and Dominic is 3 feet from the truck. Meira and Tamara are sitting on opposite endpoints of another chord containing the truck. Meira is 8 feet from the truck. How many feet, to the nearest tenth of a foot, is Tamara from the truck? Draw a diagram to support your answer

6. In the diagram below of circle O , secant \overline{AB} intersects circle O at D , secant \overline{AOC} intersects circle O at E , $AE = 4$, $AB = 12$, and $DB = 6$. What is the length of \overline{OC} ?

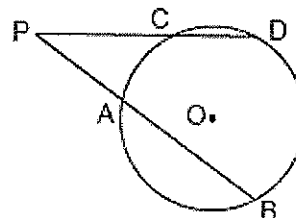


(Not drawn to scale)

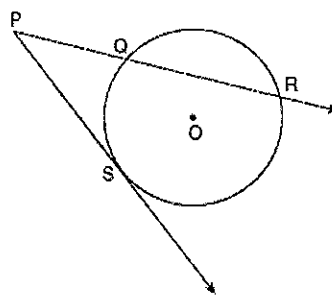
7. In the accompanying diagram of circle O , secant \overline{CBA} and \overline{CED} intersect at C .
 If $AB = 9$, $BC = 3$, and $DC = 9$, find EC .



8. In the accompanying diagram, \overline{PAB} and \overline{PCD} are secants drawn to circle O , $PA = 8$, $PB = 20$, and $PD = 16$. What is PC ?

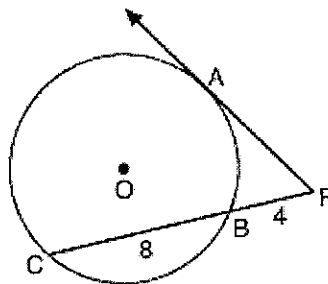


9. In the diagram below, \overline{PS} is a tangent to circle O at point S , \overline{PQR} is a secant, $PS = x$, $PQ = 3$, and $PR = x + 18$. What is the length of \overline{PS} ?



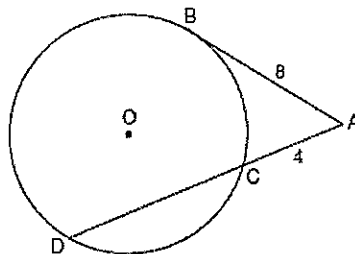
(Not drawn to scale)

10. In the accompanying diagram, \overline{PA} is tangent to circle O at A , \overline{PBC} is a secant, $PB = 4$, and $BC = 8$. What is the length of \overline{PA} in simplest radical form?

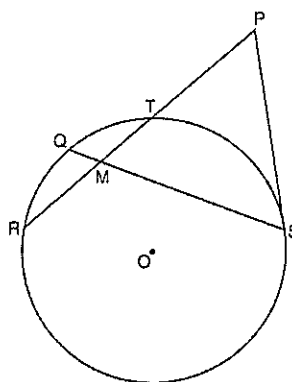


11. In the diagram below, tangent \overline{AB} and secant \overline{ACD} are drawn to circle O from an external point A , $AB = 8$, and $AC = 4$.

What is the length of \overline{CD} ?



12. In the diagram below of circle O , chords \overline{RT} and \overline{QS} intersect at M . Secant \overline{PTR} and tangent \overline{PS} are drawn to circle O . The length of \overline{RM} is two more than the length of \overline{TM} , $QM = 2$, $SM = 12$, and $PT = 8$. Find the length of \overline{RT} . Find the length of \overline{PS} .

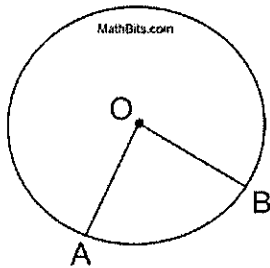


Unit 13 Lesson 4

Central and Inscribed Angles

1. Central Angle

A central angle is an angle formed by two radii with the vertex at the center of the circle.



Central Angle = Intercepted Arc
 $m\angle AOB = m\widehat{AB}$

Example 1

Given: $\odot A$ with $\widehat{BC} = 56^\circ$ & $\angle CAD = 78^\circ$

Find:

$\angle CAB =$ _____

$\widehat{DC} =$ _____

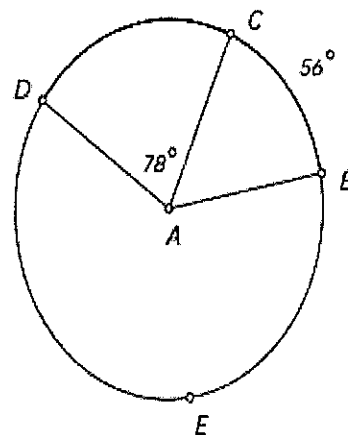
$\widehat{DB} =$ _____

$\angle DAB =$ _____

$\widehat{BED} =$ _____

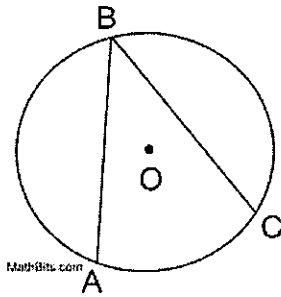
$\widehat{BEC} =$ _____

$\widehat{CED} =$ _____



2. Inscribed Angle

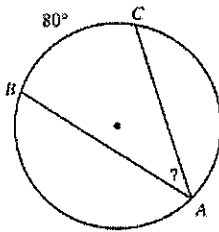
An inscribed angle is an angle with its vertex "on" the circle, formed by two intersecting chords.



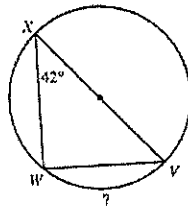
$$\text{Inscribed Angle} = \frac{1}{2} \text{ Intercepted Arc}$$

$$m\angle ABC = \frac{1}{2} m\widehat{AC}$$

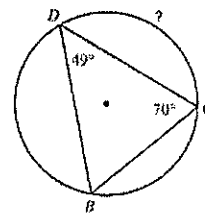
Example 2



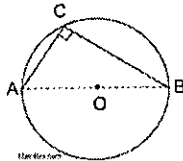
Example 3



Example 4



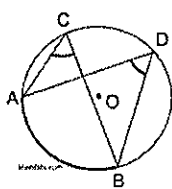
THEOREM: An angle inscribed in a semicircle is a right angle. (Called Thales Theorem.)



$$\begin{aligned} m\angle ACB &= \frac{1}{2}(m\widehat{AB}) \\ &= \frac{1}{2}(180^\circ) \\ &= 90^\circ \end{aligned}$$

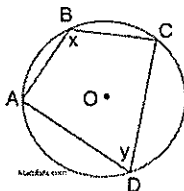
Additional theorems that result from INSCRIBED ANGLES

THEOREM: In a circle, inscribed angles that intercept the same arc are congruent.



$$\begin{aligned} m\angle C &= \frac{1}{2}(m\widehat{AB}) \\ m\angle D &= \frac{1}{2}(m\widehat{AB}) \\ m\angle C &= m\angle D \\ \angle C &\cong \angle D \end{aligned}$$

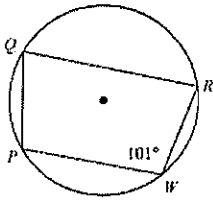
THEOREM: The opposite angles in a cyclic quadrilateral are supplementary.



A quadrilateral inscribed in a circle is called a cyclic quadrilateral.

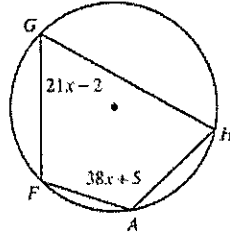
Example 5

Find $m\widehat{PQR}$



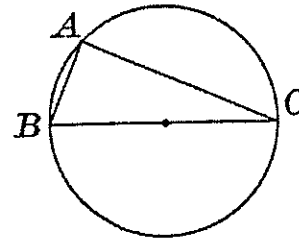
Example 6

Find $m\widehat{FGH}$



Example 7

find $m\angle BAC$

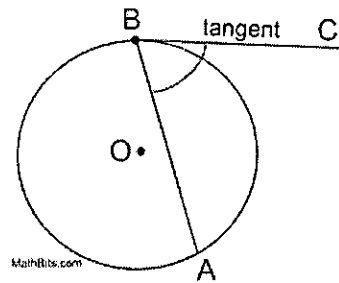


3. Tangent Chord Angle

An angle formed by an intersecting tangent and chord has its vertex "on" the circle.

$$\text{Tangent Chord Angle} = \frac{1}{2} \text{ Intercepted Arc}$$

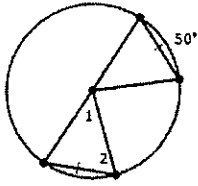
$$m\angle ABC = \frac{1}{2}(m\widehat{AB})$$



Example 8

Determine the requested value(s).

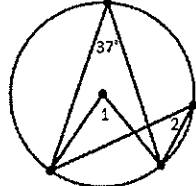
a)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

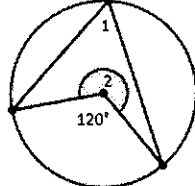
b)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

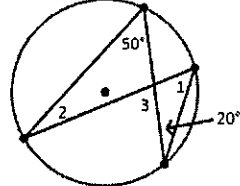
c)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

d)

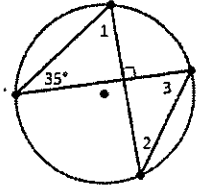


$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

$$m\angle 3 = \underline{\hspace{2cm}}$$

e)

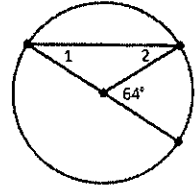


$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

$$m\angle 3 = \underline{\hspace{2cm}}$$

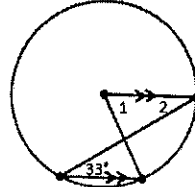
f)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

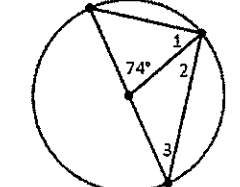
h)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

i)



$$m\angle 1 = \underline{\hspace{2cm}}$$

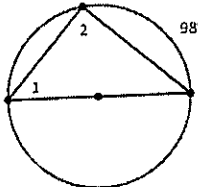
$$m\angle 2 = \underline{\hspace{2cm}}$$

$$m\angle 3 = \underline{\hspace{2cm}}$$

Example 9

Determine the requested value(s).

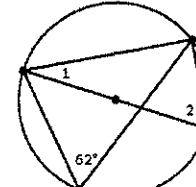
a)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

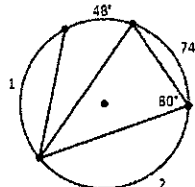
b)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

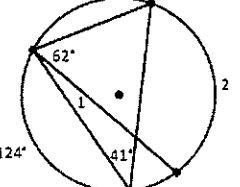
c)



$$m\hat{1} = \underline{\hspace{2cm}}$$

$$m\hat{2} = \underline{\hspace{2cm}}$$

d)



$$m\angle 1 = \underline{\hspace{2cm}}$$

$$m\angle 2 = \underline{\hspace{2cm}}$$

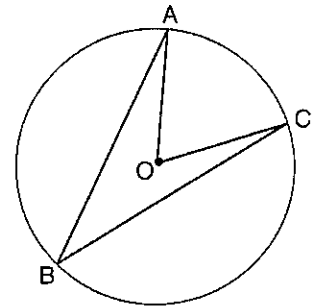
Date _____

Geometry CC

Unit 13 Lesson 4 Central and Inscribed Angles HW

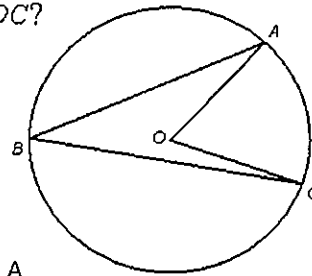
1 Circle O with $\angle AOC$ and $\angle ABC$ is shown in the diagram below. What is the ratio of $m\angle AOC$ to $m\angle ABC$?

- 1) 1:1
- 2) 2:1
- 3) 3:1
- 4) 1:2



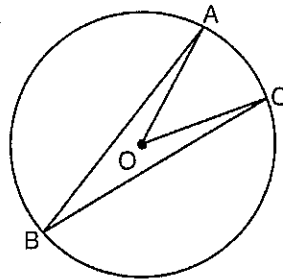
2 In the diagram below, circle O has $m\angle ABC = z$. What is $m\angle AOC$?

- 1) z
- 2) $2z$
- 3) $\frac{1}{2}z$
- 4) z^2



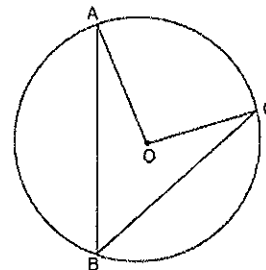
3 In the diagram below of circle O , $m\angle ABC = 24$. What is the $m\angle AOC$?

- 1) 12
- 2) 24
- 3) 48
- 4) 60



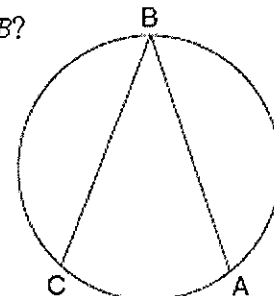
4 In the accompanying diagram of circle O , \overline{AB} and \overline{BC} are chords and $m\angle AOC = 96$. What is $m\angle ABC$?

- 1) 32
- 2) 48
- 3) 96
- 4) 192



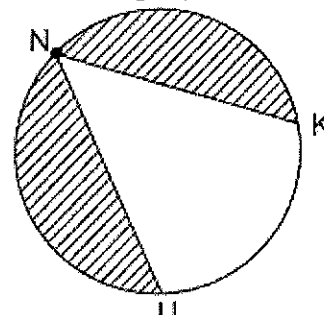
5 The new corporate logo created by the design engineers at Magic Motors is shown in the accompanying diagram. If chords \overline{BA} and \overline{BC} are congruent and $m\widehat{BC} = 140$, what is $m\angle B$?

- 1) 40
- 2) 80
- 3) 140
- 4) 280

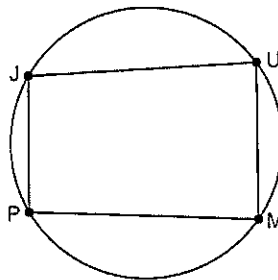


6 The NUK Energy Company is designing a new logo, as shown in the accompanying diagram, with $m\widehat{NK} = 130$ and $m\widehat{NK} = m\widehat{NU}$. What is the measure of $\angle KNU$?

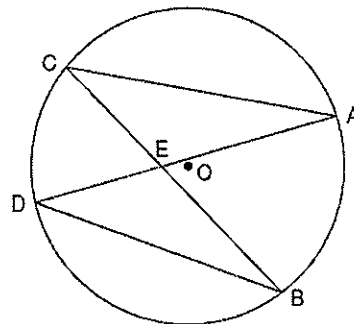
- 1) 50°
- 2) 65°
- 3) 80°
- 4) 100°



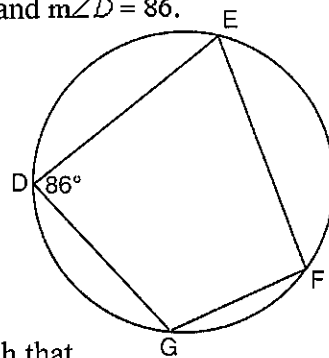
- 7 In the diagram below, quadrilateral $JUMP$ is inscribed in a circle. Opposite angles J and M must be
- 1) right
 - 2) complementary
 - 3) congruent
 - 4) supplementary



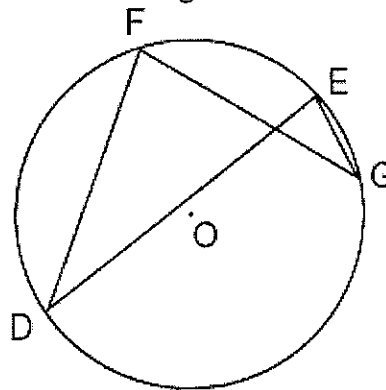
- 8 In the diagram below of circle O , chords \overline{AD} and \overline{BC} intersect at E . Which relationship must be true?
- 1) $\triangle CAE \cong \triangle DBE$
 - 2) $\triangle AEC \sim \triangle BED$
 - 3) $\angle ACB \cong \angle CBD$
 - 4) $\widehat{CA} \cong \widehat{DB}$



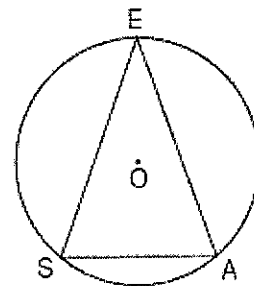
- 9 As shown in the diagram below, quadrilateral $DEFG$ is inscribed in a circle and $m\angle D = 86$. Determine and state $m\widehat{GFE}$. Determine and state $m\angle F$.



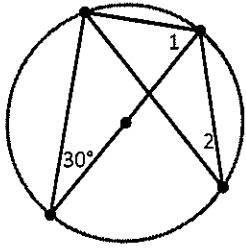
- 10 In the diagram below of circle O , chords \overline{DF} , \overline{DE} , \overline{FG} , and \overline{EG} are drawn such that $m\widehat{DF} : m\widehat{FE} : m\widehat{EG} : m\widehat{GD} = 5 : 2 : 1 : 7$. Identify one pair of inscribed angles that are congruent to each other and give their measure.



- 11 A machine part consists of a circular wheel with an inscribed triangular plate, as shown in the accompanying diagram. If $\overline{SE} \cong \overline{EA}$, $SE = 10$, and $m\widehat{SE} = 140$, find the length of \overline{SA} to the nearest tenth.

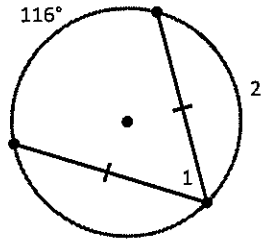


12. j)



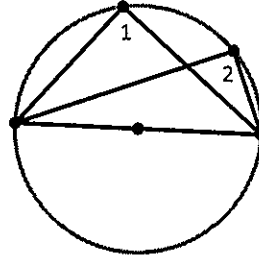
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$

k)



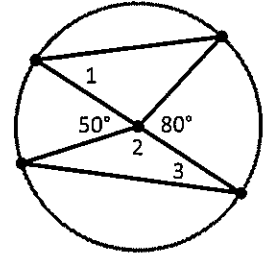
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\hat{2} = \underline{\hspace{2cm}}$

l)



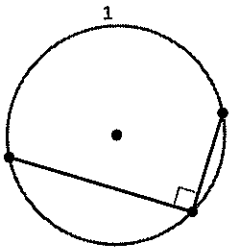
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$

m)



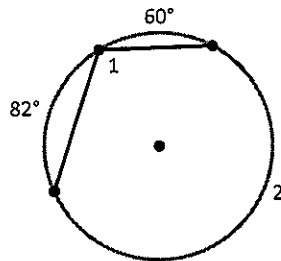
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$
 $m\angle 3 = \underline{\hspace{2cm}}$

n)



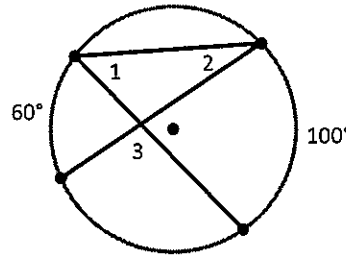
$m\hat{1} = \underline{\hspace{2cm}}$

o)



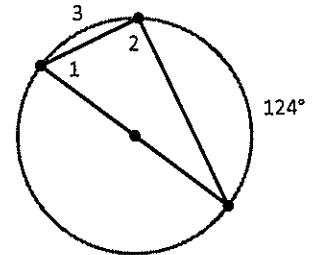
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\hat{2} = \underline{\hspace{2cm}}$

p)



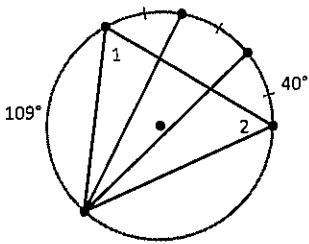
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$
 $m\angle 3 = \underline{\hspace{2cm}}$

q)



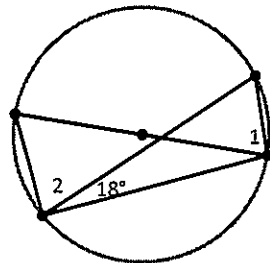
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$
 $m\hat{3} = \underline{\hspace{2cm}}$

13.e)



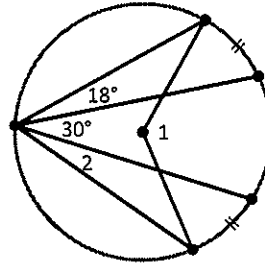
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$

f)



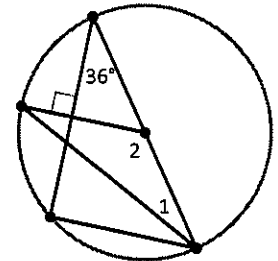
$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$

g)



$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$

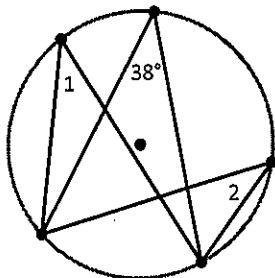
h)



$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$

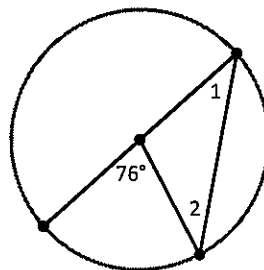
14. Determine the requested value(s).

a)



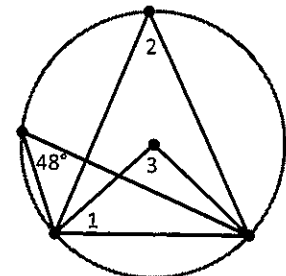
$m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$

b)



$m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$

c)

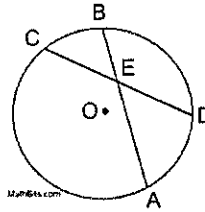


$m\angle 1 = \underline{\hspace{2cm}}$
 $m\angle 2 = \underline{\hspace{2cm}}$
 $m\angle 3 = \underline{\hspace{2cm}}$

Unit 13 Lesson 5

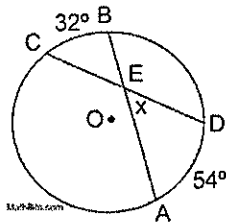
Angles Formed by Intersecting Chords

When two chords intersect inside a circle, four angles are formed. At the point of intersection, two sets of congruent vertical angles are formed in the corners of the X that appears.



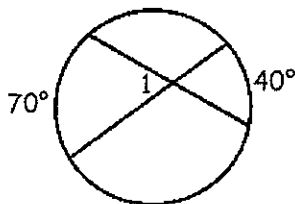
Angle Formed by Two Chords
 $= \frac{1}{2} (\text{SUM of Intercepted Arcs})$
 $m\angle AED = \frac{1}{2} (m\widehat{AD} + m\widehat{CB})$

Example Solve for x

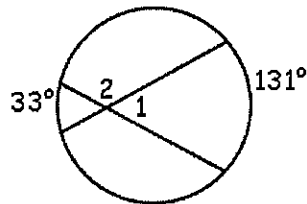


Practice: Find all unknown variables

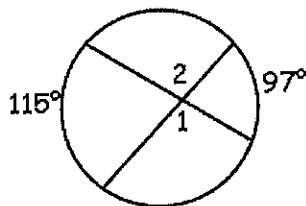
1. $m\angle 1 = \underline{\hspace{2cm}}$



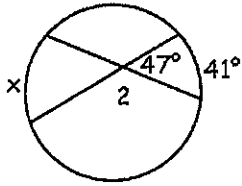
2. $m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$



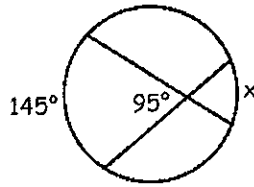
3. $m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$



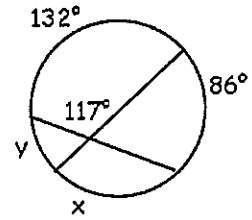
4. $m\angle 2 = \underline{\hspace{1cm}}$ $x = \underline{\hspace{1cm}}$



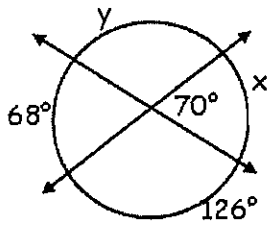
5. $x = \underline{\hspace{1cm}}$



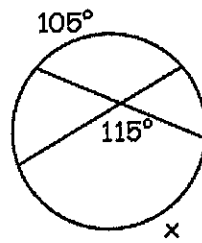
6. $x = \underline{\hspace{1cm}}$ $y = \underline{\hspace{1cm}}$



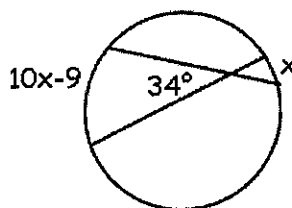
7. $x = \underline{\hspace{1cm}}$ $y = \underline{\hspace{1cm}}$



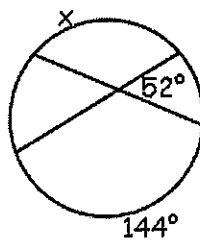
8. $x = \underline{\hspace{1cm}}$



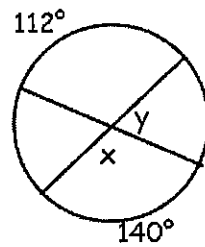
9. $x = \underline{\hspace{1cm}}$



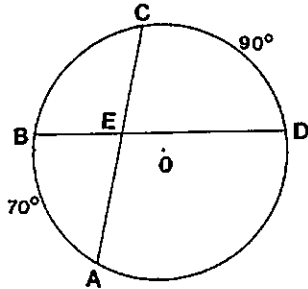
10. $x = \underline{\hspace{1cm}}$



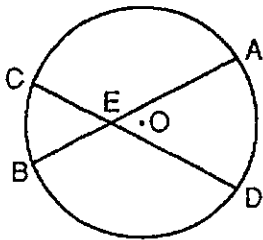
11. $x = \underline{\hspace{1cm}}$ $y = \underline{\hspace{1cm}}$



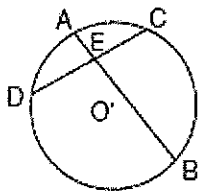
1. In the accompanying diagram, \overline{AC} and \overline{BD} are chords of circle O and intersect at E . If $m\widehat{AB} = 70$ and $m\widehat{CD} = 90$, find $m\angle BEA$.



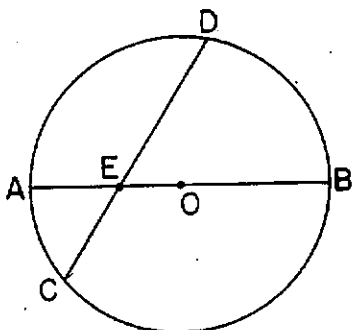
2. In the accompanying diagram, chords \overline{AB} and \overline{CD} intersect at E . If $m\widehat{AD} = 70$ and $m\widehat{BC} = 40$, find $m\angle AED$.



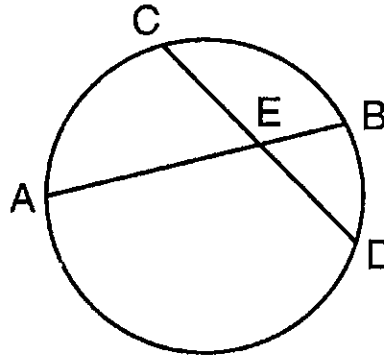
3. In the accompanying diagram of circle O , chords \overline{AB} and \overline{CD} intersect at E , $m\widehat{AC} = 50$, and $m\widehat{BD} = 150$. Find $m\angle AED$.



4. In the accompanying diagram, \overline{AB} is a diameter of circle O and chord \overline{CD} intersects diameter \overline{AB} at E . If $m\widehat{AD} = 100$ and $m\widehat{AC} = 40$, find $m\angle DEB$.

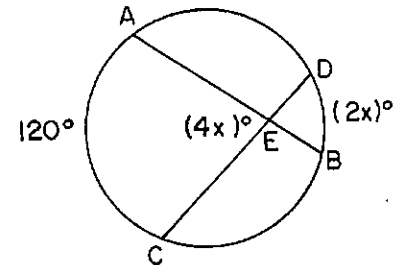


5. In the accompanying diagram, chords \overline{AB} and \overline{CD} intersect at E . If $m\widehat{AC} = 75$ and $m\widehat{DB} = 45$, find $m\angle AED$.



6. In the diagram below, chords \overline{AB} and \overline{CD} intersect at E . If $m\angle AEC = 4x$, $m\widehat{AC} = 120$, and $m\widehat{DB} = 2x$, what is the value of x ?

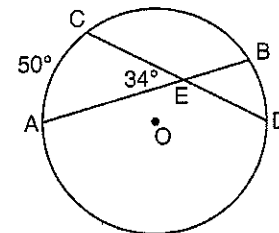
- 1) 12
- 2) 20
- 3) 30
- 4) 60



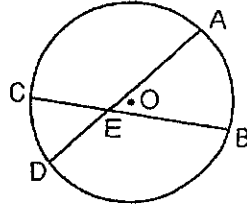
7. In the diagram below of circle O , chords \overline{AB} and \overline{CD} intersect at E .

If $m\angle AEC = 34$ and $m\widehat{AC} = 50$, what is $m\widehat{DB}$?

- 1) 16
- 2) 18
- 3) 68
- 4) 118

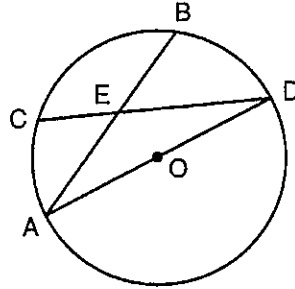


8. In the accompanying diagram of circle O ,
 $m\widehat{AB} = 64$ and $m\angle AEB = 52$.

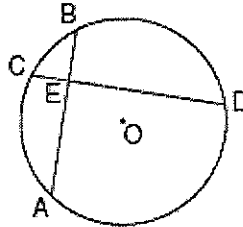


What is the measure of \widehat{CD} ?

- 1) 104°
 - 2) 80°
 - 3) 52°
 - 4) 40°
9. In the accompanying figure of circle O , chords \overline{AB} and \overline{CD} intersect at E and \overline{AD} is a diameter. If $m\widehat{CB} = 82$, find $m\angle AED$.

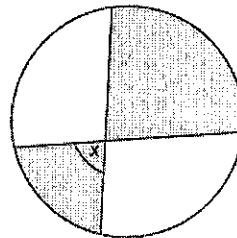


10. In the accompanying diagram of circle O , chords \overline{AB} and \overline{CD} intersect at E and $m\widehat{AC} : m\widehat{CB} : m\widehat{BD} : m\widehat{DA} = 4 : 2 : 6 : 8$.



What is $m\angle DEB$?

- 1) 36
 - 2) 90
 - 3) 100
 - 4) 126
11. The accompanying diagram shows a child's spin toy that is constructed from two chords intersecting in a circle. The curved edge of the larger shaded section is one-quarter of the circumference of the circle, and the curved edge of the smaller shaded section is one-fifth of the circumference of the circle. What is the measure of angle x ?



Unit 13 Lesson 6

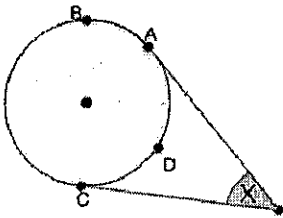
Angle Formed Outside of Circle by Intersection:

"Two Tangents" or "Two Secants" or a "Tangent and a Secant".

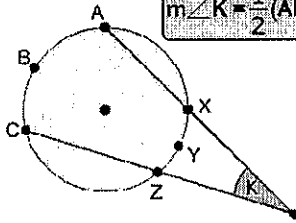
The formulas for all THREE of these situations are the same:

$$\text{Angle Formed Outside} = \frac{1}{2} (\text{DIFFERENCE of Intercepted Arcs})$$

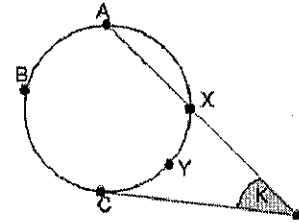
$$m\angle X = \frac{1}{2} (\widehat{ABC} - \widehat{GDA})$$



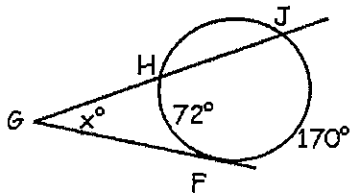
$$m\angle K = \frac{1}{2} (\widehat{ABC} - \widehat{XYZ})$$



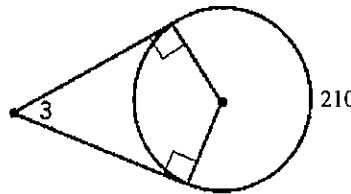
$$m\angle K = \frac{1}{2} (\widehat{ABC} - \widehat{XYC})$$



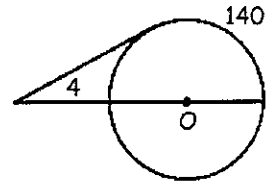
1. $x = \underline{\hspace{2cm}}$



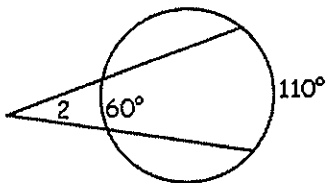
2. $m\angle 3 = \underline{\hspace{2cm}}$



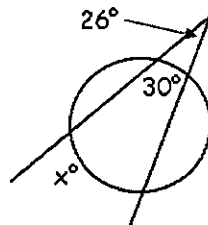
3. $m\angle 4 = \underline{\hspace{2cm}}$



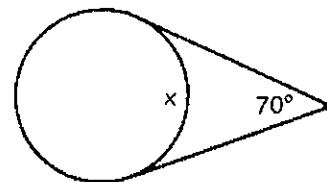
4. $m\angle 2 = \underline{\hspace{2cm}}$



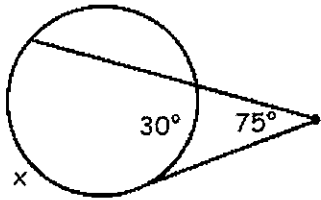
5. $x = \underline{\hspace{2cm}}$



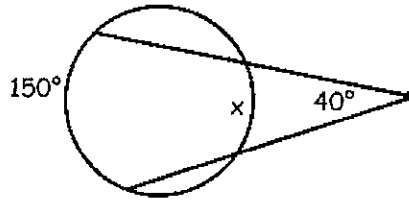
6. $x = \underline{\hspace{2cm}}$



7. $x =$ _____

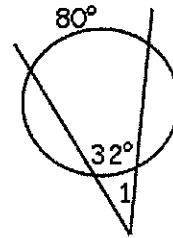
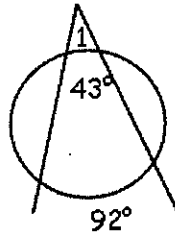
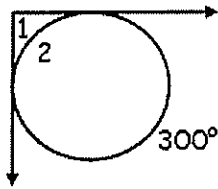


8. $x =$ _____



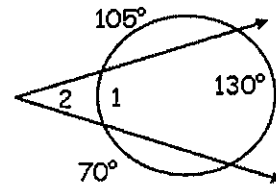
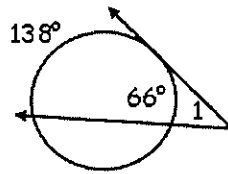
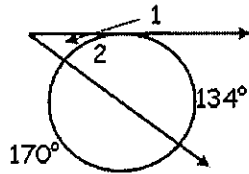
9. $m\angle 1 =$ _____ $mArc2 =$ _____ 10. $m\angle 1 =$ _____

11. $m\angle 1 =$ _____



12. $m\angle 1 =$ _____ $mArc2 =$ _____ 13. $m\angle 1 =$ _____

14. $mArc1 =$ _____ $m\angle 2 =$ _____

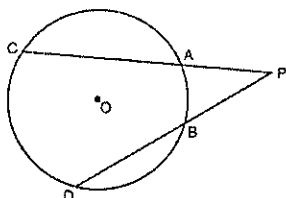


Unit 13 lesson 6 Angles formed outside of a circle HW

1. In the diagram below of circle O , \overline{PAC} and \overline{PBD} are secants.

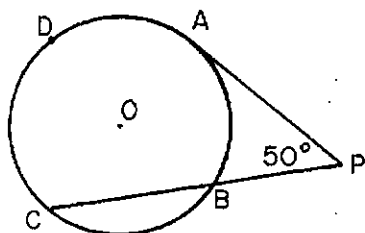
If $m\widehat{CD} = 70$ and $m\widehat{AB} = 20$, what is the degree measure of $\angle P$?

- 1) 25
- 2) 35
- 3) 45
- 4) 50



2. In the accompanying diagram, tangent \overline{PA} and secant \overline{PBC} are drawn to circle O . If $m\widehat{ADC}$ is twice $m\widehat{AB}$ and $m\angle P$ is 50, what is $m\widehat{AB}$?

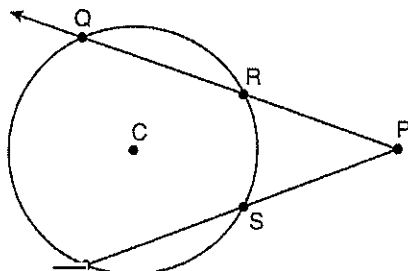
- 1) 25
- 2) 50
- 3) 100
- 4) 200



3. In the diagram below of circle C , $m\widehat{QT} = 140$, and $m\angle P = 40$.

What is $m\widehat{RS}$?

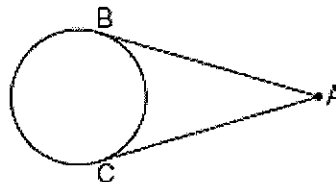
- 1) 50
- 2) 60
- 3) 90
- 4) 110



4. In circle O , \overline{PA} and \overline{PB} are tangent to the circle from point P . If the ratio of the measure of major arc AB to the measure of minor arc AB is 5:1, then $m\angle P$ is

- 1) 60
- 2) 90
- 3) 120
- 4) 180

5. The accompanying diagram shows two lengths of wire attached to a wheel, so that \overline{AB} and \overline{AC} are tangent to the wheel. If the major arc \widehat{BC} has a measure of 220° , find the number of degrees in $m\angle A$.

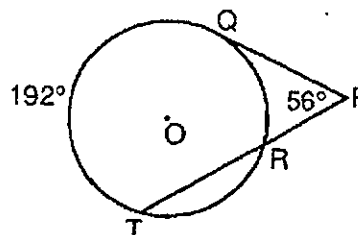


6. Two tangents \overline{PA} and \overline{PB} are drawn to circle O from an external point P . If the measure of major arc \widehat{AB} is 250° , find the measure of $\angle P$.

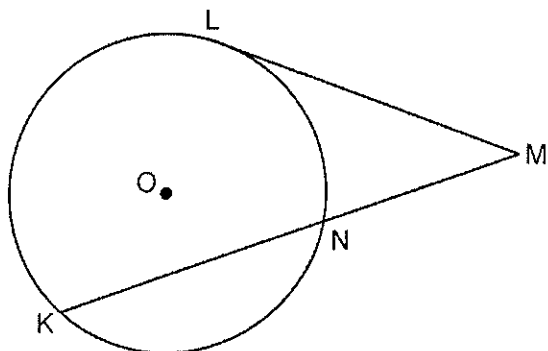
7. Two tangents to a circle from an external point intercept a major arc of 300° . Find the number of degrees in the angle formed by the two tangents.

8. Tangents \overline{PA} and \overline{PB} are drawn from point P to the same circle. The major arc intercepted by the tangents is three times the minor arc. Find $m\angle APB$.

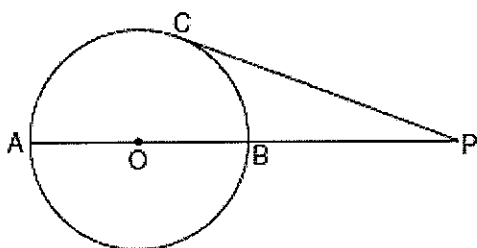
9. In the accompanying diagram, \overline{PQ} is tangent to circle O at Q and \overline{PRT} is a secant. If $m\angle P = 56$ and $m\widehat{QT} = 192$, find $m\widehat{QR}$.



10. In the diagram below, tangent \overline{ML} and secant \overline{MKN} are drawn to circle O . The ratio $m\widehat{LN} : m\widehat{NK} : m\widehat{KL}$ is $3:4:5$. Find $m\angle LMK$.



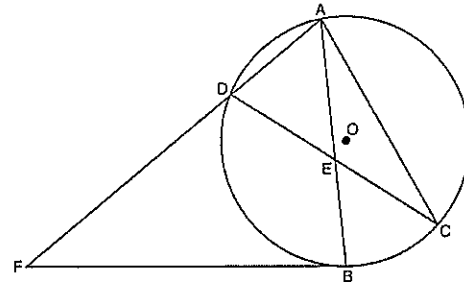
11. In the accompanying diagram of circle O , diameter \overline{AOB} is extended through B to external point P , tangent \overline{PC} is drawn to point C on the circle, and $m\widehat{AC} : m\widehat{BC} = 7:2$. Find $m\angle CPA$.



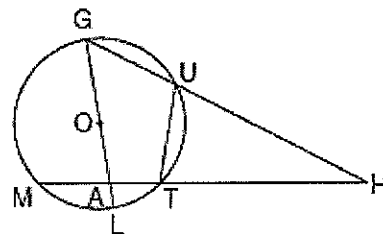
(Not drawn to scale)

12. Point P lies outside circle O , which has a diameter of \overline{AOC} . The angle formed by tangent \overline{PA} and secant \overline{PBC} measures 30° . Sketch the conditions given above and find the number of degrees in the measure of minor arc CB .

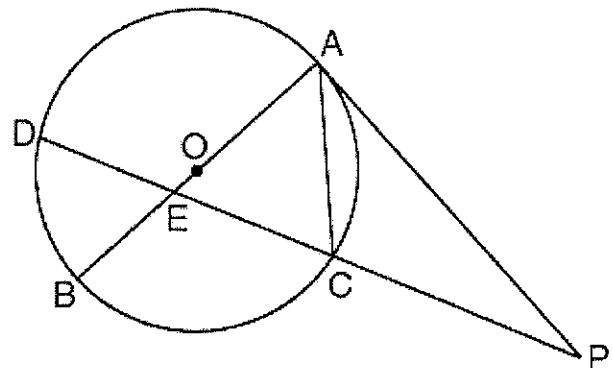
1. Chords \overline{AB} and \overline{CD} intersect at E in circle O , as shown in the diagram below. Secant \overline{FDA} and tangent \overline{FB} are drawn to circle O from external point F and chord \overline{AC} is drawn. The $m\widehat{DA} = 56$, $m\widehat{DB} = 112$, and the ratio of $m\widehat{AC} : m\widehat{CB} = 3 : 1$. Determine $m\angle CEB$. Determine $m\angle F$. Determine $m\angle DAC$.



2. Given circle O with diameter \overline{GOAL} ; secants \overline{HUG} and \overline{HTAM} intersect at point H ; $m\widehat{GM} : m\widehat{ML} : m\widehat{LT} = 7 : 3 : 2$; and chord $\overline{GU} \cong \text{chord } \overline{UT}$. Find the ratio of $m\angle UGL$ to $m\angle H$.

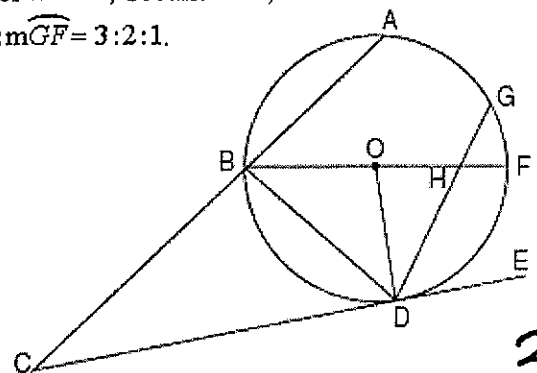


3. In the accompanying diagram, \overline{PA} is tangent to circle O at A , chord \overline{AC} and secant \overline{PCED} are drawn, and chords \overline{AOB} and \overline{CD} intersect at E . If $m\widehat{AD} = 130$ and $m\angle BAC = 50$, find $m\angle P$, $m\angle BEC$, and $m\angle PCA$.



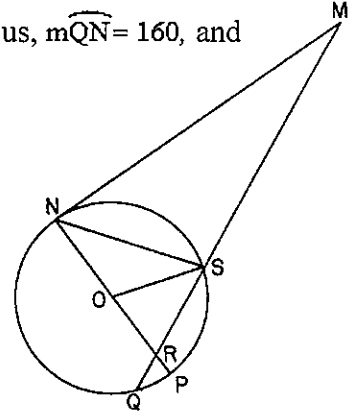
4. In the accompanying diagram, circle O has radius \overline{OD} , diameter \overline{BOHF} , secant \overline{CBA} , and chords \overline{DHG} and \overline{BD} ; \overline{CE} is tangent to circle O at D ; $m\widehat{DF} = 80$; and $m\widehat{BA} : m\widehat{AG} : m\widehat{GF} = 3 : 2 : 1$.

Find $m\widehat{GF}$, $m\angle BHD$, $m\angle BDG$, $m\angle GDE$, $m\angle C$, and $m\angle BOD$.



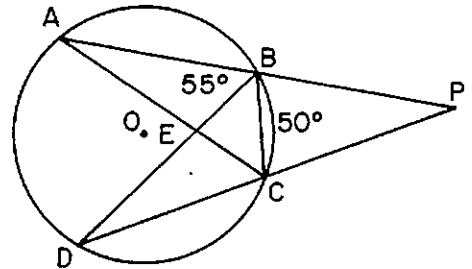
5. In circle O , \overline{MN} is a tangent, \overline{NP} is a diameter, \overline{MQ} is a secant, \overline{OS} is a radius, $m\widehat{QN} = 160$, and $m\angle PNS = 40$.

Find $m\widehat{QP}$, $m\widehat{PS}$, $m\angle QRP$, $m\angle NOS$, and $m\angle M$



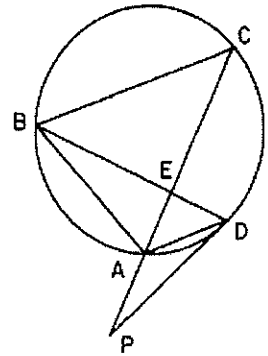
6. In the accompanying diagram of circle O , \overline{PBA} and \overline{PCD} are secants, chords \overline{AC} and \overline{BD} intersect at E , $\overline{BA} \cong \overline{CD}$, chord \overline{BC} is drawn, $m\angle ABD = 55$, and $m\widehat{BC} = 50$.

Find: $m\angle ACD$, $m\angle P$, $m\angle DBC$, $m\angle AED$, $m\angle PCB$.



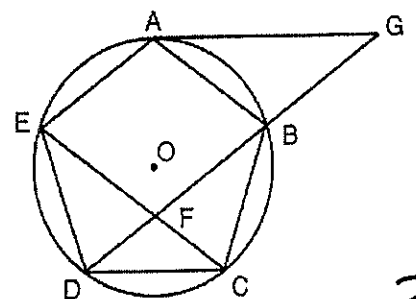
7. In the accompanying diagram, $\triangle ABC$ is isosceles with $\overline{CB} \cong \overline{CA}$, $m\angle DAC = 45$, $m\widehat{BC} = 135$, \overline{PD} is tangent to circle O at D , \overline{PAC} is a secant, and chords \overline{BD} and \overline{AC} intersect at E .

Find: $m\widehat{AD}$, $m\widehat{AB}$, $m\angle P$, $m\angle ADP$, $m\angle BEC$

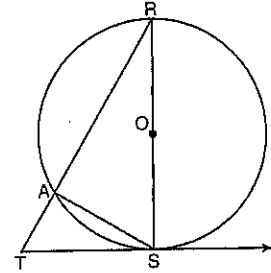


8. In the accompanying diagram, regular pentagon $ABCDE$ is inscribed in circle O , chords \overline{EC} and \overline{DB} intersect at F , chord \overline{DB} is extended to G , and tangent \overline{GA} is drawn.

Find: $m\angle BDE$, $m\angle BFC$, $m\angle AGD$



1. In the diagram of circle O below, diameter \overline{RS} , chord \overline{AS} , tangent \overrightarrow{TS} , and secant \overline{TAR} are drawn. Complete the following proof to show $(RS)^2 = RA \cdot RT$



Statements

Reasons

1. circle O , diameter \overline{RS} , chord \overline{AS} , tangent \overrightarrow{TS} , and secant \overline{TAR}

1. Given

2. $\overline{RS} \perp \overrightarrow{TS}$

2. _____

3. $\angle RST$ is a right angle

3. \perp lines form right angles

4. $\angle RAS$ is a right angle

4. _____

5. $\angle RST \cong \angle RAS$

5. _____

6. $\angle R \cong \angle R$

6. Reflexive property

7. $\triangle RST \sim \triangle RAS$

7. _____

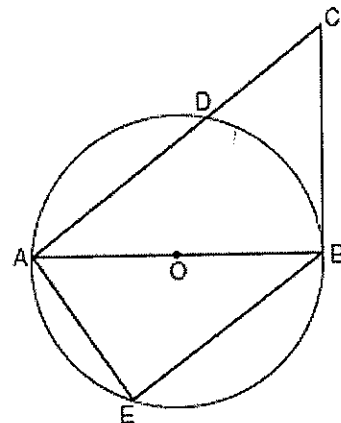
8. $\frac{RS}{RA} = \frac{RT}{RS}$

8. _____

9. $(RS)^2 = RA \cdot RT$

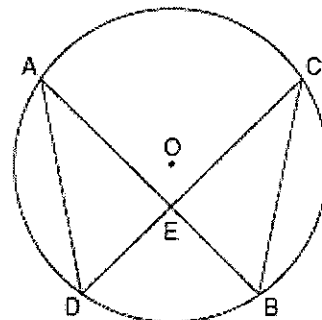
9. _____

2. In the accompanying diagram of circle O , diameter \overline{AOB} is drawn, tangent \overline{CB} is drawn to the circle at B , E is a point on the circle, and $\overline{BE} \parallel \overline{ADC}$. Prove: $\triangle ABE \sim \triangle CAB$

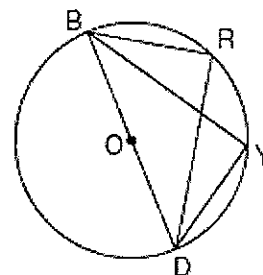


3. Given: chords \overline{AB} and \overline{CD} of circle O intersect at E , an interior point of circle O ; chords \overline{AD} and \overline{CB} are drawn.

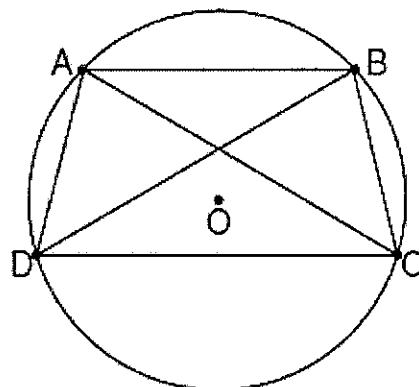
Prove: $(AE)(EB) = (CE)(ED)$



4. In the accompanying diagram, $m\widehat{BR} = 70$, $m\widehat{YD} = 70$, and \overline{BOD} is the diameter of circle O . Write an explanation or a proof that shows $\triangle RBD$ and $\triangle YBD$ are congruent.

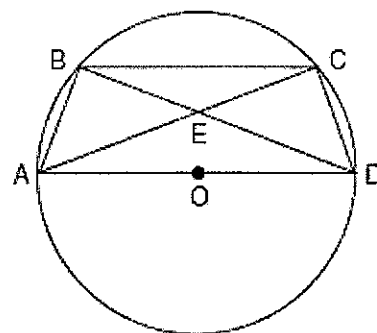


5. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O , $\overline{AB} \parallel \overline{DC}$, and diagonals \overline{AC} and \overline{BD} are drawn. Prove that $\triangle ACD \cong \triangle BDC$.



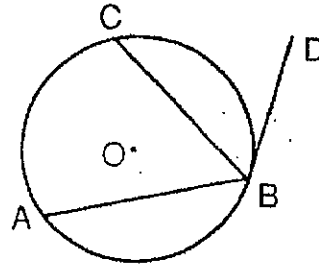
6. In the accompanying diagram of circle O , \overline{AD} is a diameter with \overline{AD} parallel to chord \overline{BC} , chords \overline{AB} and \overline{CD} are drawn, and chords \overline{BD} and \overline{AC} intersect at E .

Prove: $\overline{BE} \cong \overline{CE}$

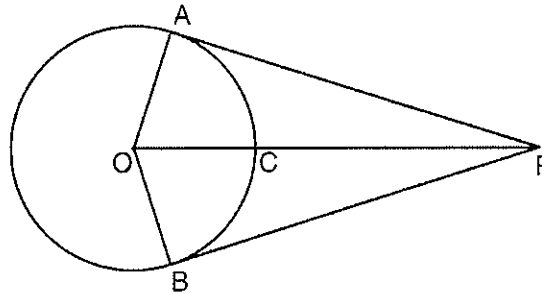


7. Given: circle O , \overline{DB} is tangent to the circle at B , \overline{BC} and \overline{BA} are chords, and C is the midpoint of \widehat{AB} .

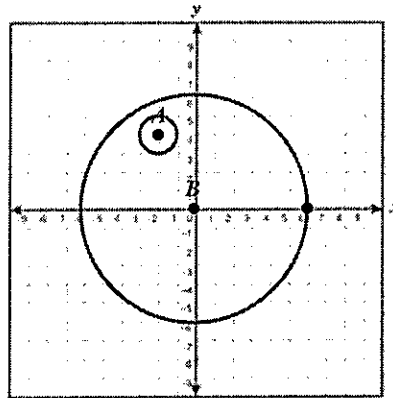
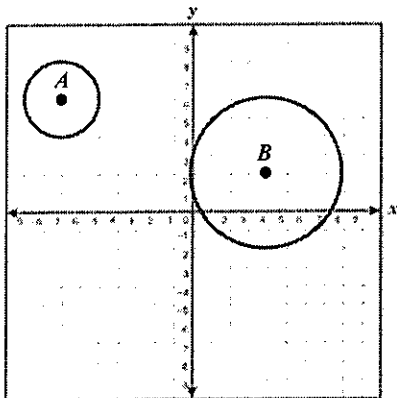
Prove: $\angle ABC \cong \angle CBD$



8. In the diagram below, \overline{PA} and \overline{PB} are tangent to circle O , \overline{OA} and \overline{OB} are radii, and \overline{OP} intersects the circle at C . Prove: $\angle AOP \cong \angle BOP$



9. Determine the translation vector and scale factor of the dilation for the following similarity transformations.

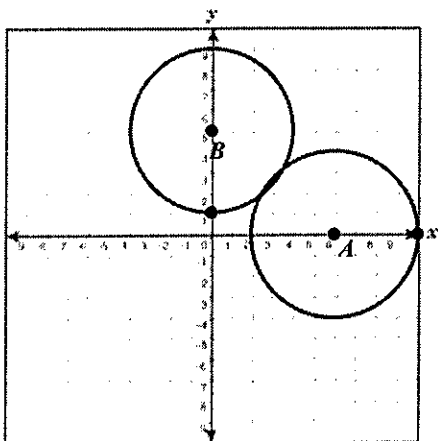


Unit 4-13 Test: Circles REVIEW QUESTIONS

1. Determine the center and radius of the given circles.

- | | | |
|------------------------------|---------------------|---------------|
| a) $(x-7)^2 + (y+10)^2 = 81$ | Center (____, ____) | Radius = ____ |
| b) $100 = (x+3)^2 + y^2$ | Center (____, ____) | Radius = ____ |
| c) $(x-9)^2 + (y+2)^2 = 1$ | Center (____, ____) | Radius = ____ |

2. Write the equation of each circle



3. Write the equation of each circle

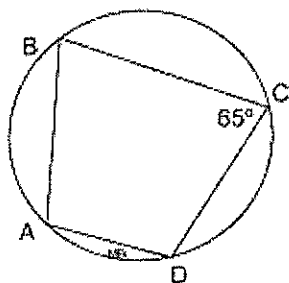
- a) Radius = 7 cm Center (-3,6)
- b) Center (1, 6) passing through (-3, 3)
- c) Endpoints of diameter are (0, -2) and (4, -4)
- d) Radius = 10 cm C (3,-3)

4. What are the center and radius of each circle

- a) $x^2 - 14x + y^2 - 2y - 50 = 0$
- b) $x^2 + y^2 + 18x + 17 = 0$

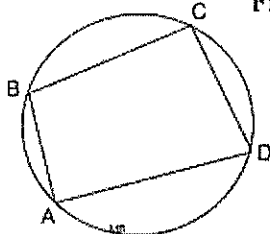
5.

a. Find $m\angle A$.

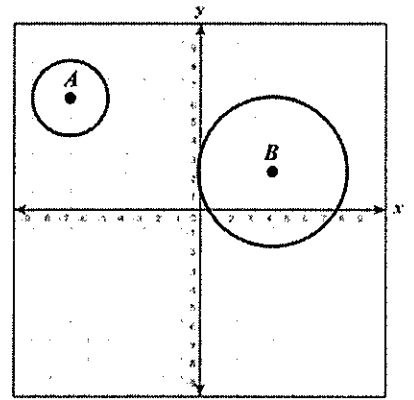


b. $m\angle B = 110^\circ$ and $m\angle C = 100^\circ$.

Find $m\widehat{BCD}$.

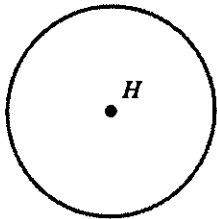


6. Determine the translation vector and scale factor of the dilation for the following similarity transformations.

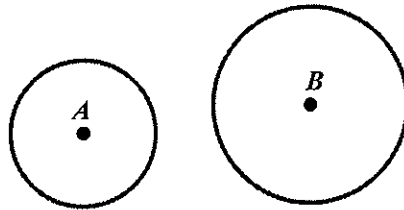


7. Draw the following relationships.

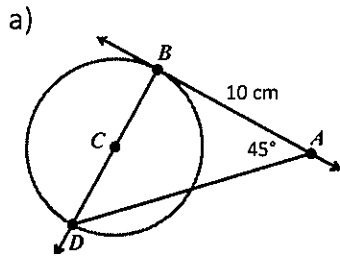
a) \overline{AB} tangent to circle H at B.



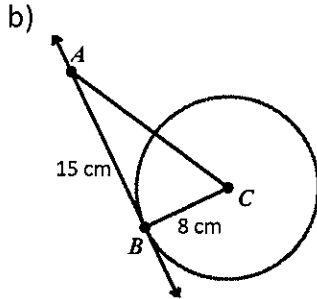
b) The external tangents of circle A and B.



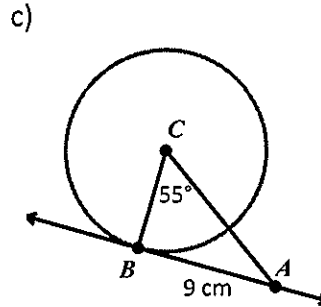
8. Solve for the missing information, given the \overline{AB} is a tangent line to circle C.



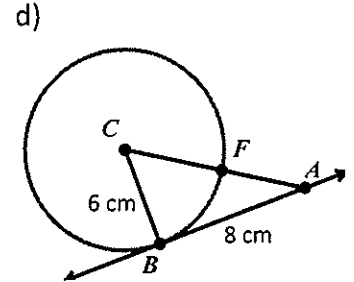
CB = _____ AC = _____



AC = _____

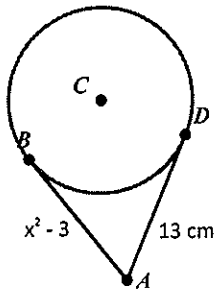


CB = _____

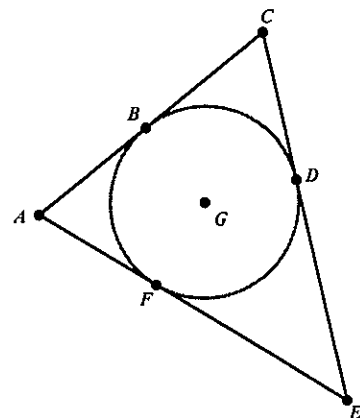


FA = _____

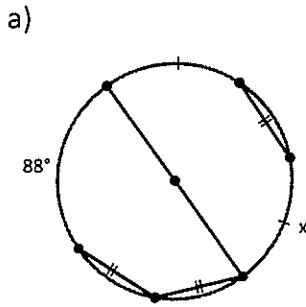
9. Solve for x (\overline{AB} and \overline{AD} are tangent lines)



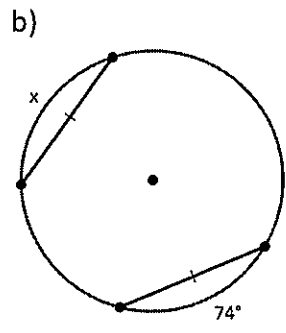
10. AC = 14 cm, CE = 16 cm and EA = 18 cm. Determine AB, CD and FE.



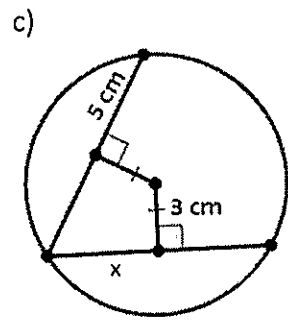
11. Solve for x.



x = _____

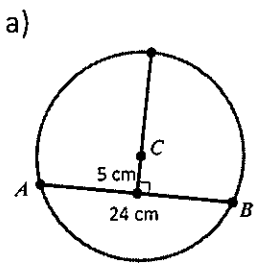


x = _____

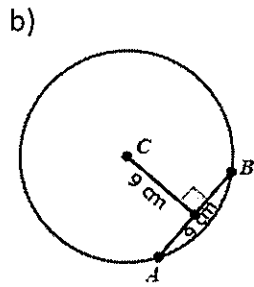


x = _____

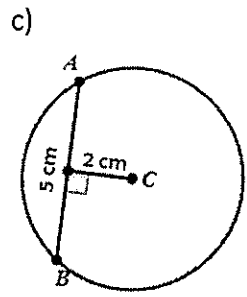
12. Determine the length of radius \overline{AC} .



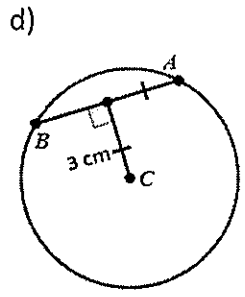
AC = _____



AC = _____ (2 dec.)

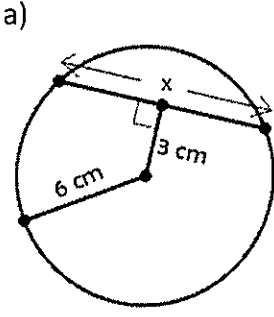


AC = _____ (2 dec.)

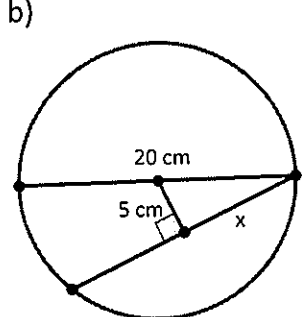


AC = _____ (E)

13. Find x in simplest radical form.

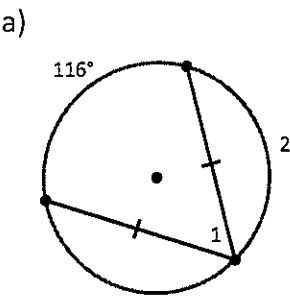


x = _____

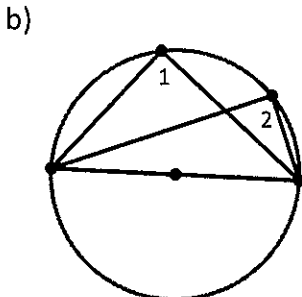


x = _____ (E)

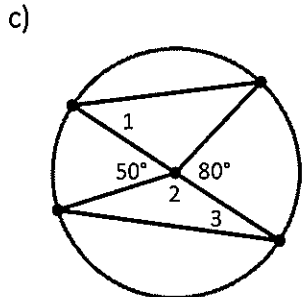
14. Find the measure of each angle or arc.



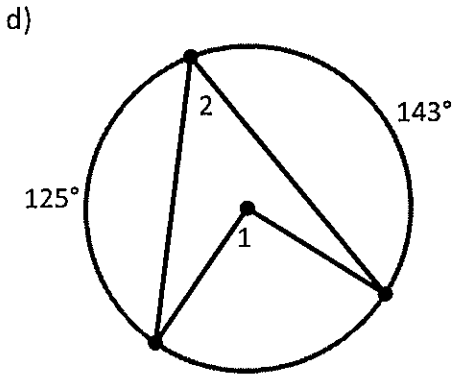
$m\angle 1 =$ _____
 $m\widehat{2} =$ _____



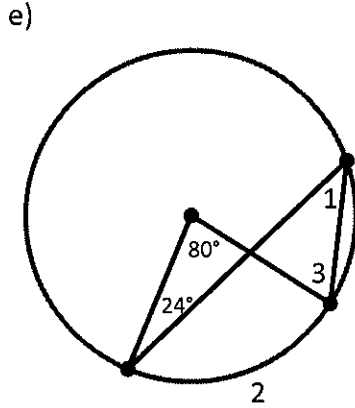
$m\angle 1 =$ _____
 $m\angle 2 =$ _____



$m\angle 1 =$ _____
 $m\angle 2 =$ _____
 $m\angle 3 =$ _____

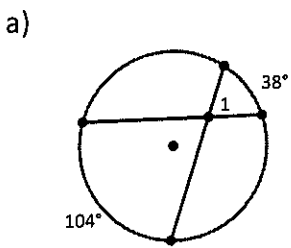


$m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$

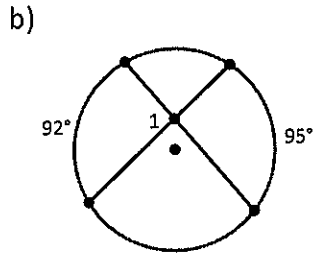


$m\angle 1 = \underline{\hspace{2cm}}$
 $m\widehat{2} = \underline{\hspace{2cm}}$
 $m\angle 3 = \underline{\hspace{2cm}}$

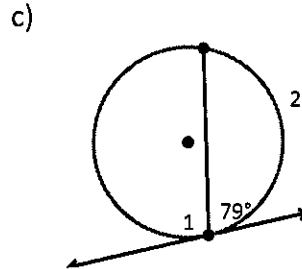
15. Determine the requested value(s). (Lines that appear to be tangent are tangent.)



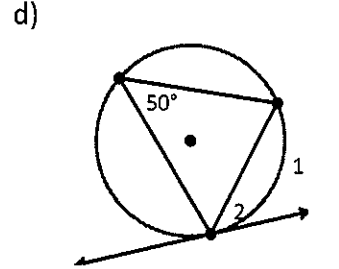
$m\angle 1 = \underline{\hspace{2cm}}$



$m\angle 1 = \underline{\hspace{2cm}}$

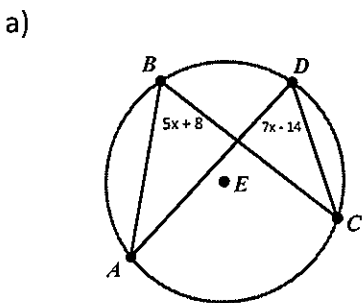


$m\angle 1 = \underline{\hspace{2cm}}$ $m\widehat{2} = \underline{\hspace{2cm}}$

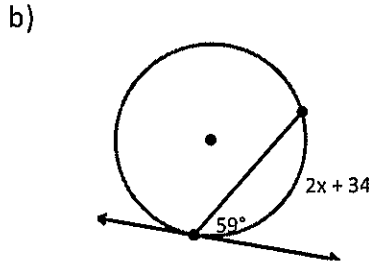


$m\widehat{1} = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$

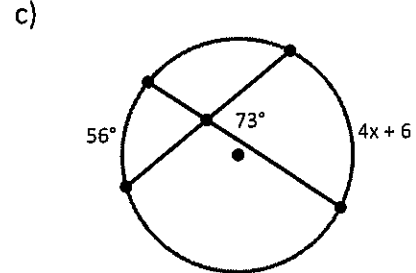
16. Determine the requested value(s). (Lines that appear to be tangent are tangent.)



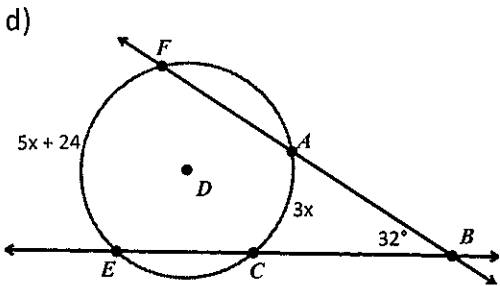
$x = \underline{\hspace{2cm}}$ $m\angle ABC = \underline{\hspace{2cm}}$



$x = \underline{\hspace{2cm}}$



$x = \underline{\hspace{2cm}}$



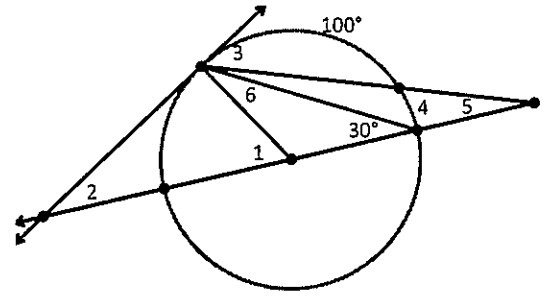
$x = \underline{\hspace{2cm}}$ $m\widehat{EF} = \underline{\hspace{2cm}}$

17. Solve for the missing values.

a) $m\angle 1 =$ _____ b) $m\angle 2 =$ _____

c) $m\angle 3 =$ _____ d) $m\hat{4} =$ _____

e) $m\angle 5 =$ _____ f) $m\angle 6 =$ _____



18. Solve for x

