

Geometry

Unit 4-12

Coordinate Geometry Proofs

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Lesson 1 Triangle Proofs

Example 1:

The vertices of $\triangle ABC$ are $A(-2, 4)$, $B(-2, 8)$ and $C(-5, 6)$.
Prove $\triangle ABC$ is isosceles.

$$CB = \sqrt{(-5 - (-2))^2 + (6 - 8)^2}$$

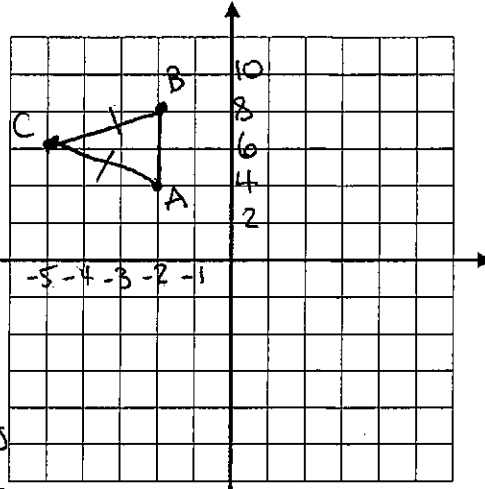
$$= \sqrt{9 + 4} = \sqrt{13}$$

$$AC = \sqrt{(-2 - (-5))^2 + (4 - 6)^2}$$

$$= \sqrt{9 + 4} = \sqrt{13}$$

\cong segments
have =
distances.

$\triangle ABC$ is an isosceles triangle because it has 2 \cong sides.



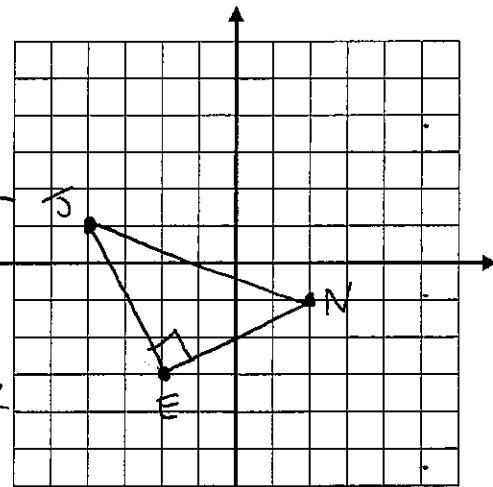
Example 2:

The vertices of $\triangle JEN$ are $J(-4, 1)$, $E(-2, -3)$ and $N(2, -1)$.
Prove $\triangle JEN$ is a right triangle.

$$m_{\overline{JE}} = \frac{1 - (-3)}{-4 - (-2)} = \frac{4}{-2} = -2$$

$$m_{\overline{EN}} = \frac{-3 - (-1)}{-2 - 2} = \frac{-2}{-4} = \frac{1}{2}$$

Perpendicular lines have neg. reciprocal slopes and they form a right \angle . $\triangle JEN$ is a right triangle because it has a right \angle , $\angle JEN$.



HW

Example 3:

Prove that $A(4,-1)$, $B(5,6)$, $C(1,3)$ is an isosceles right triangle.

$$BC = \sqrt{(5-1)^2 + (6-3)^2}$$

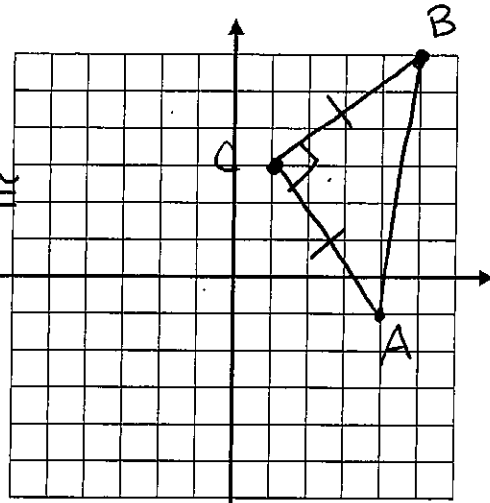
$$= \sqrt{16+9} = \sqrt{25} = 5$$

$$AC = \sqrt{(4-1)^2 + (-1-3)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

$$m_{\overline{BC}} = \frac{6-3}{5-1} = \frac{3}{4}$$

$$m_{\overline{AC}} = \frac{-1-3}{4-1} = -\frac{4}{3}$$



$\triangle ABC$ is an isosceles right \triangle because it has 2 \cong sides, $\overline{BC} \cong \overline{AC}$, and a right \angle because $\overline{BC} \perp \overline{AC}$ (their slopes are neg. reciprocals)

Example 4:

The coordinates of $\triangle ABC$ are $A(0,0)$, $B(2,6)$, and $C(4,2)$. Using coordinate geometry, prove that if the midpoints of sides \overline{AB} and \overline{AC} are joined, the segment formed is parallel to the third side and equal to one-half the length of the third side.

$$\text{midpt } \overline{AB} = \left(\frac{0+2}{2}, \frac{0+6}{2} \right)$$

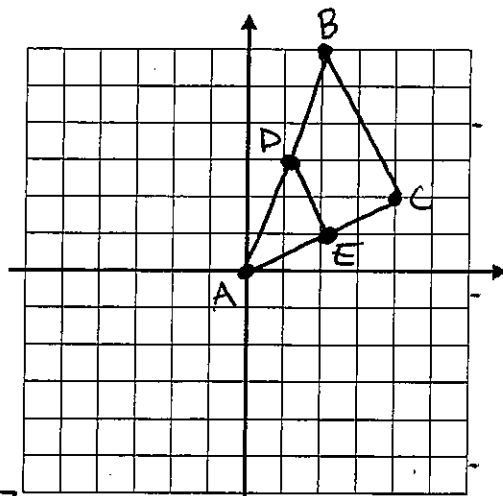
$$D = (1, 3)$$

$$\text{midpt } \overline{AC} = \left(\frac{0+4}{2}, \frac{0+2}{2} \right)$$

$$E = (2, 1)$$

$$m_{\overline{BC}} = \frac{6-2}{2-4} = \frac{4}{-2} = -2$$

$$m_{\overline{DE}} = \frac{3-1}{1-2} = \frac{2}{-1} = -2$$



The midpt of \overline{AB} is $D(1,3)$ and the midpt of \overline{AC} is $E(2,1)$. The segment joining

the midpoints, \overline{DE} , is \parallel to the 3rd side \overline{BC} b/c 2

HW

Example 5:

The vertices of $\triangle NYS$ are $N(-2,-1)$, $Y(0,10)$, and $S(10,5)$. The coordinates of point T are $(4,2)$.

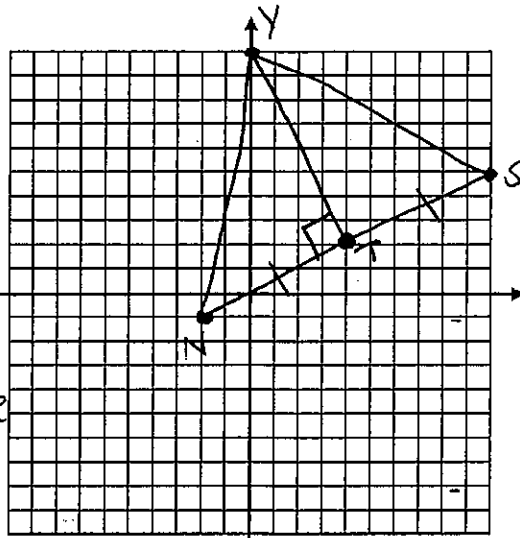
(a) Prove that \overline{YT} is a median.

(b) Prove that \overline{YT} is an altitude.

(c) Find the area of $\triangle NYS$.

$$\begin{aligned} \text{a) } m_{\text{pt } \overline{NS}} &= \left(\frac{-2+10}{2}, \frac{-1+5}{2} \right) \\ &= (4, 2) = T \end{aligned}$$

\overline{YT} is a median because T is the midpoint of \overline{NS}



$$\text{b) } m_{\overline{YT}} = \frac{10-2}{0-4} = \frac{8}{-4} = -2$$

$$m_{\overline{NS}} = \frac{5-(-1)}{10-(-2)} = \frac{6}{12} = \frac{1}{2}$$

Perpendicular lines have neg. reciprocal slopes and form rt. \angle 's. \overline{YT} is an altitude because it is \perp to \overline{NS} and they share point T .

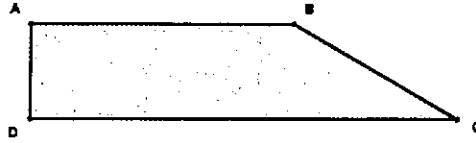
$$\text{c) } h = \overline{YT} = \sqrt{(0-4)^2 + (10-2)^2} = \sqrt{16+64} = \sqrt{80} = h$$

$$b = \overline{NS} = \sqrt{(-2-10)^2 + (-1-5)^2} = \sqrt{144+36} = \sqrt{180} = b$$

$$A_{\triangle} = \frac{1}{2}bh = \frac{1}{2}(\sqrt{180})(\sqrt{80}) = \frac{1}{2}\sqrt{14400} = \frac{1}{2}(120) = 60$$

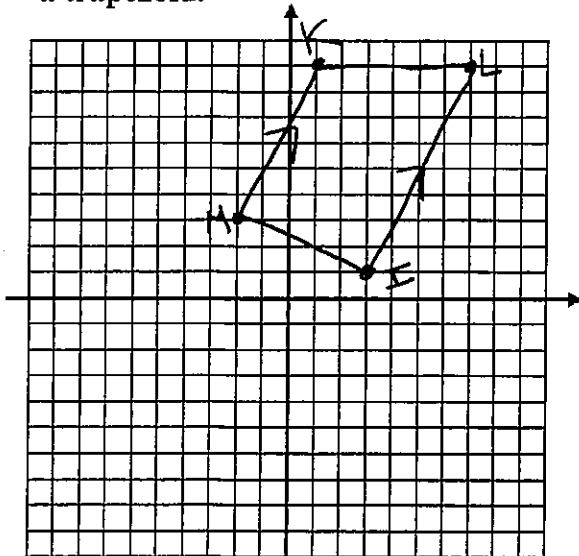
Unit 12 Lesson 2

Trapezoid and Isosceles Trapezoid



Example 1

Prove that quadrilateral MILK with the vertices M(-2,3), I(3, 1), L(7, 9), and K(1, 9) is a trapezoid.



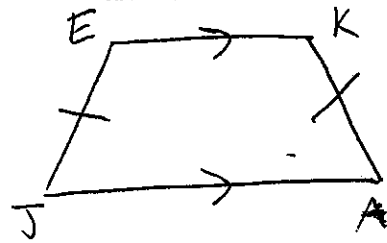
$$m_{\overline{MK}} = \frac{9-3}{1-(-2)} = \frac{6}{3} = 2$$

$$m_{\overline{IL}} = \frac{9-1}{7-3} = \frac{8}{4} = 2$$

Quad. MILK is a trapezoid because it has one pair of \parallel sides. $\overline{MK} \parallel \overline{IL}$ because they have equal slopes.

Example 2

Quadrilateral JAKE has coordinates
 $J(0, 3a)$, $A(3a, 3a)$, $K(4a, 0)$ and $E(-a, 0)$.



Prove by coordinate geometry that quadrilateral JAKE is an isosceles trapezoid.

$$m\overline{JA} = \frac{3a-3a}{0-3a} = \frac{0}{-3a} = 0$$

$$m\overline{KE} = \frac{0-0}{4a-a} = \frac{0}{3a} = 0$$

Parallel lines have = slopes + $m\overline{JA} = m\overline{KE}$ so $\overline{JA} \parallel \overline{KE}$. \cong seg. have = distances + $EJ = AK$, so $\overline{EJ} \cong \overline{AK}$. Quad.

$$EJ = \sqrt{(0-(-a))^2 + (3a-0)^2} = \sqrt{a^2 + 9a^2} = \sqrt{10a^2}$$

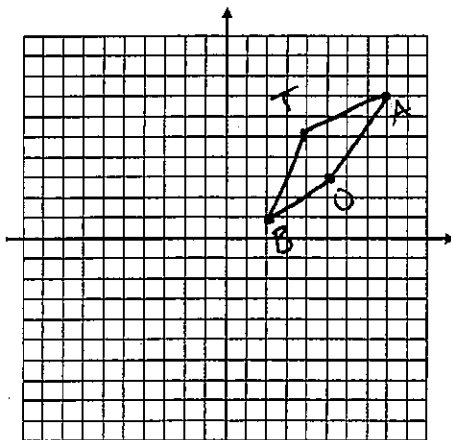
$$AK = \sqrt{(3a-4a)^2 + (3a-0)^2} = \sqrt{a^2 + 9a^2} = \sqrt{10a^2}$$

JAKE is an isos. trap. because it has 1 pair of \parallel sides + non- \parallel sides are \cong .

Example 3

Quadrilateral BOAT has coordinates
 $B(2, 1)$, $O(6, 3)$, $A(8, 7)$ and $T(4, 5)$.

Prove by coordinate geometry that the diagonals of BOAT bisect each other.



$$\text{midpt } \overline{BA} = \left(\frac{2+8}{2}, \frac{1+7}{2} \right) = (5, 4)$$

$$\text{midpt } \overline{TO} = \left(\frac{6+4}{2}, \frac{3+5}{2} \right) = (5, 4)$$

since midpt $\overline{BA} = \text{midpt } \overline{TO}$ + segments that bisect each other have the same midpts, then the diagonals of BOAT bisect each other.

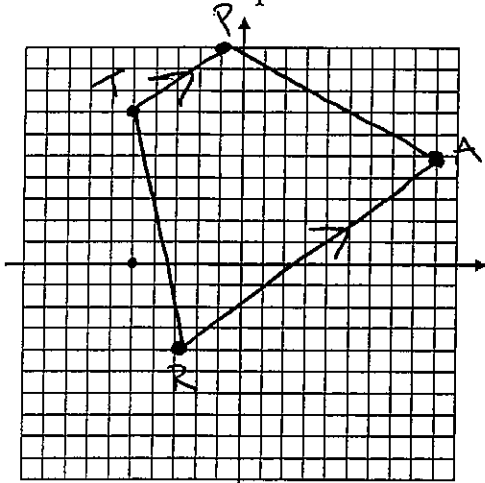
HW

Example 4

Given quadrilateral TRAP with coordinates T(-5, 7), R(-3, -4) and A(9, 5).

Determine and state coordinates of P that would make TRAP a trapezoid.

Then prove, using coordinate geometry that TRAP is a trapezoid.



$$m_{\overline{RA}} = \frac{-4-5}{-3-9} = \frac{-9}{-12} = \frac{3}{4}$$

$$P(-1, 10)$$

$$m_{\overline{TP}} = \frac{7-10}{-5-(-1)} = \frac{-3}{-4} = \frac{3}{4}$$

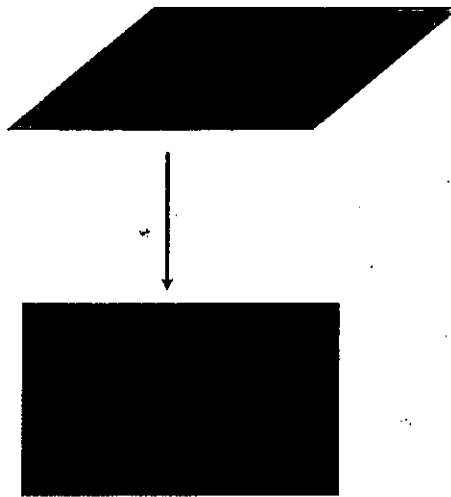
*Count w/ slope from T

||

The $m_{\overline{RA}} = m_{\overline{TP}}$ + parallel lines have equal slopes. TRAP is a trapezoid because it has one pair of || sides.

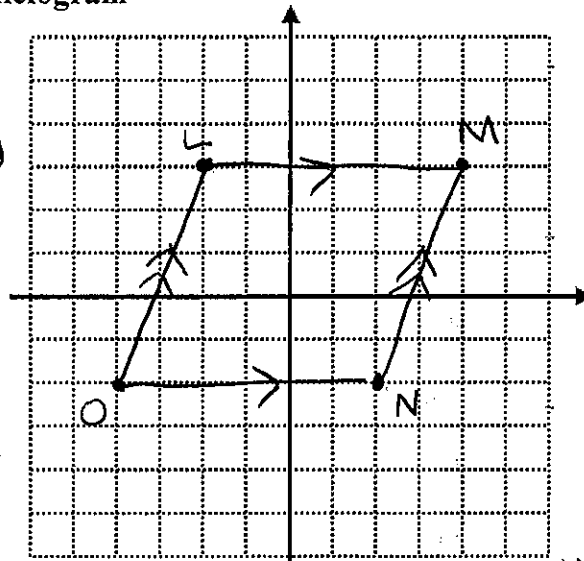
Unit 12 Lesson 3

Parallelogram and Rectangle



Example 1 Prove that the quadrilateral with the coordinates $L(-2,3)$, $M(4,3)$, $N(2,-2)$ and $O(-4,-2)$ is a parallelogram

$$\begin{aligned}
 m_{\overline{LM}} &= \frac{3-3}{-2-4} = 0 \\
 m_{\overline{ON}} &= \frac{-2-2}{-4-2} = 0 \\
 m_{\overline{LO}} &= \frac{3-2}{-2-4} = \frac{5}{2} \\
 m_{\overline{MN}} &= \frac{3-2}{4-2} = \frac{5}{2}
 \end{aligned}$$



The $m_{\overline{LM}} = m_{\overline{ON}}$ & $m_{\overline{LO}} = m_{\overline{MN}}$ & parallel lines have = slopes so $\overline{LM} \parallel \overline{ON}$ & $\overline{LO} \parallel \overline{MN}$. Quad. LMNO is a parallelogram because it has 2 pairs of \parallel sides.

Example 2

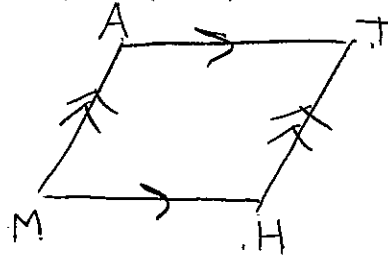
Prove that the quadrilateral with the coordinates $M(0, 0)$, $A(r, t)$, $T(s, t)$ and $H(s - r, 0)$ is a parallelogram.

$$m_{\overline{HT}} = \frac{t-0}{s-(s-r)} = \frac{t}{r}$$

$$m_{\overline{MA}} = \frac{0-t}{0-r} = \frac{t}{r} \quad \parallel$$

$$m_{\overline{MH}} = \frac{0-0}{0-(s-r)} = 0$$

$$m_{\overline{AT}} = \frac{t-t}{r-s} = 0 \quad \parallel$$



Since $m_{\overline{HT}} = m_{\overline{MA}}$ & $m_{\overline{MH}} = m_{\overline{AT}}$ & parallel lines have = slopes, then $\overline{HT} \parallel \overline{MA}$ & $\overline{MH} \parallel \overline{AT}$. $MATH$ is a parallelogram b/c it has 2 pairs of \parallel sides.

Example 3

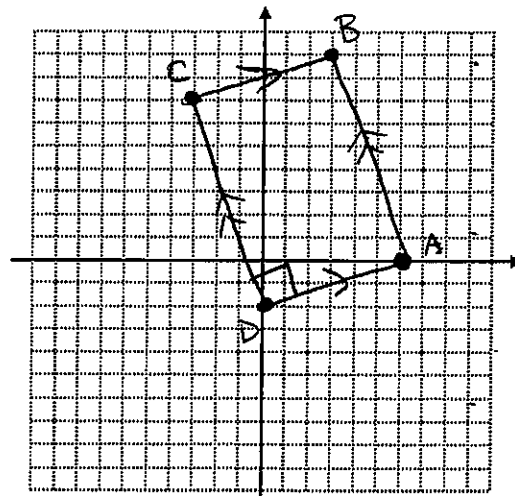
Quadrilateral ABCD has vertices $A(6, 0)$, $B(3, 9)$, $C(-3, 7)$ and $D(0, -2)$. Prove that ABCD is a rectangle.

$$m_{\overline{BC}} = \frac{9-7}{3-(-3)} = \frac{2}{6} = \frac{1}{3}$$

$$m_{\overline{AD}} = \frac{0-(-2)}{6-0} = \frac{2}{6} = \frac{1}{3} \quad \parallel$$

$$m_{\overline{CD}} = \frac{7-(-2)}{-3-0} = \frac{9}{-3} = -3$$

$$m_{\overline{AB}} = \frac{0-9}{6-3} = \frac{-9}{3} = -3 \quad \parallel$$



Since $m_{\overline{BC}} = m_{\overline{AD}}$ & $m_{\overline{CD}} = m_{\overline{AB}}$ & parallel lines have = slopes, then $\overline{BC} \parallel \overline{AD}$ & $\overline{CD} \parallel \overline{AB}$. Since $m_{\overline{AD}}$ & $m_{\overline{CD}}$ are (-) rec. & \perp lines have (-) rec.

slopes, then $\overline{AD} \perp \overline{CD}$. Quad. ABCD is a rectangle b/c it has 2 pairs of \parallel sides & a rt. \angle .

HW

Example 4

Prove that quadrilateral RATS is a rectangle.

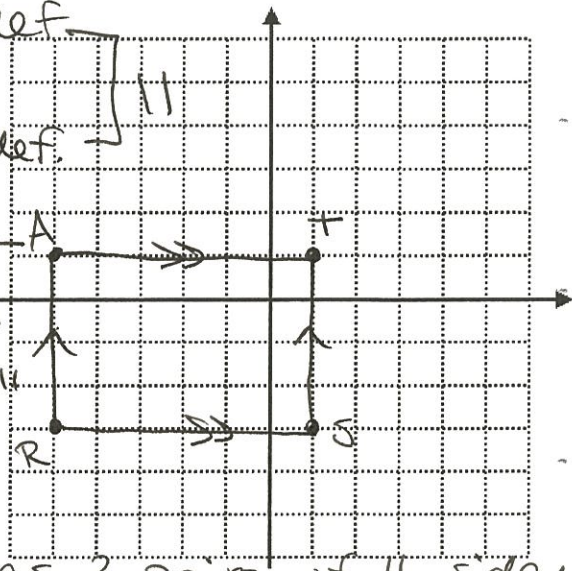
R(-5, -3) A(-5, 1) T(1, 1) S(1, -3)

$$m_{\overline{RA}} = \frac{1 - (-3)}{-5 - (-5)} = \frac{4}{0} = \text{undef.}$$

$$m_{\overline{TS}} = \frac{-3 - 1}{1 - 1} = \frac{-4}{0} = \text{undef.}$$

$$m_{\overline{AT}} = \frac{1 - 1}{-5 - 1} = \frac{0}{-6} = 0$$

$$m_{\overline{RS}} = \frac{-3 - (-3)}{-5 - (-1)} = \frac{0}{-6} = 0$$

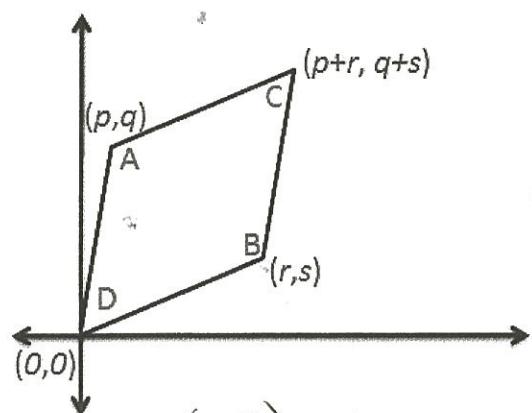


Quad. RATS is a rectangle b/c it has 2 pairs of \parallel sides & a rt. \angle

~~HW~~

Example 5

Is quadrilateral ABCD is a rectangle? Prove it.



$$m_{\overline{AC}} = \frac{q - (q+s)}{p - (p+r)} = \frac{s}{-r}$$

$$m_{\overline{AD}} = \frac{q - 0}{p - 0} = \frac{q}{p}$$

$$m_{\overline{BC}} = \frac{s - (q+s)}{r - (p+r)} = \frac{-q}{-p} = \frac{q}{p}$$

$$m_{\overline{BD}} = \frac{s - 0}{r - 0} = \frac{s}{r}$$

Since none of the slopes are neg. recip. then \overline{AC} is not \perp to \overline{AD} . Quad. ABCD is not a rectangle b/c it does not have a

rt \angle

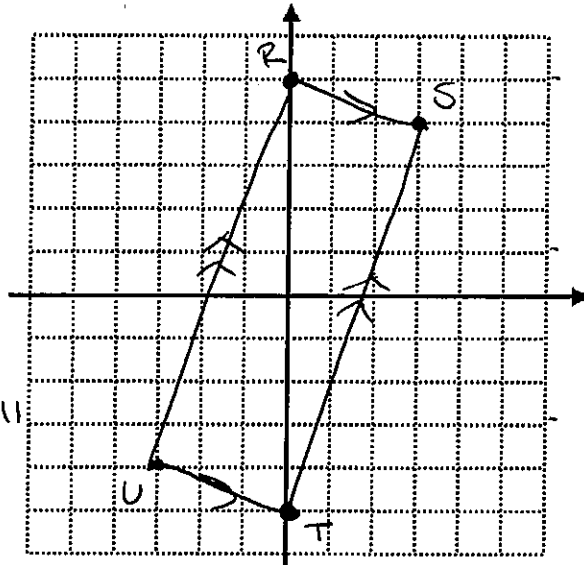
HW

Example 6

Prove that the quadrilateral with coordinates $R(0,5)$, $S(3,4)$, $T(0,-5)$ and $U(-3,-4)$ is a parallelogram.

$$\begin{aligned} m_{\overline{RS}} &= \frac{5-4}{0-3} = \frac{1}{-3} \\ m_{\overline{TU}} &= \frac{-5-4}{0-3} = \frac{-1}{3} \end{aligned} \quad \parallel$$

$$\begin{aligned} m_{\overline{RU}} &= \frac{5-4}{0-3} = \frac{9}{3} = 3 \\ m_{\overline{ST}} &= \frac{4-5}{3-0} = \frac{9}{3} = 3 \end{aligned} \quad \parallel$$

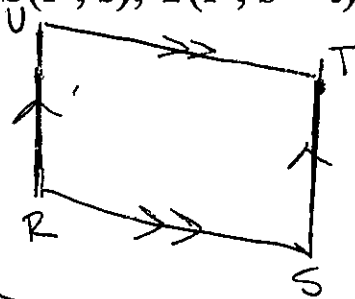


Quad. RSTU is a parallelogram b/c it has 2 pairs of \parallel sides ($\overline{RS} \parallel \overline{TU}$ & $\overline{RU} \parallel \overline{ST}$)

HW

Example 7

Prove that the quadrilateral with the coordinates $R(0,0)$, $S(r,s)$, $T(r,s+t)$ and $U(0,t)$ is a parallelogram.



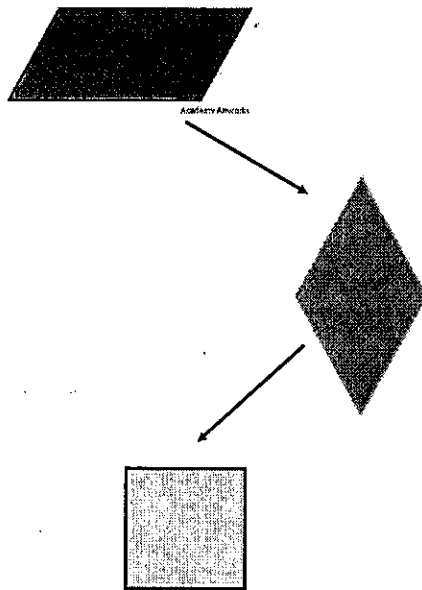
$$\begin{aligned} m_{\overline{RS}} &= \frac{s-0}{r-0} = \frac{s}{r} \\ m_{\overline{TU}} &= \frac{t-(s+t)}{0-r} = \frac{-s}{-r} = \frac{s}{r} \end{aligned} \quad \parallel$$

$$\begin{aligned} m_{\overline{ST}} &= \frac{s-(s+t)}{r-r} = \frac{-t}{0} = \text{undefined} \\ m_{\overline{RU}} &= \frac{t-0}{0-0} = \frac{t}{0} = \text{undefined} \end{aligned} \quad \parallel$$

Quad. RSTU is a parallelogram b/c it has 2 pairs of \parallel sides.

Unit 12 Lesson 4

Rhombus and Square



Example 1

Prove that a quadrilateral with the vertices

A(-2,3), B(2,6), C(7,6) and D(3,3)

is a rhombus.

$$m_{\overline{BD}} = \frac{6-3}{2-3} = \frac{3}{-1} = -3$$

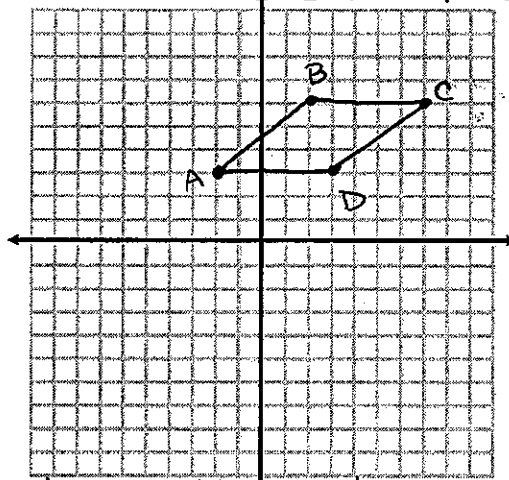
$$m_{\overline{AC}} = \frac{3-6}{-2-7} = \frac{-3}{-9} = \frac{1}{3}$$

$$m_{\overline{AB}} = \frac{3-6}{-2-2} = \frac{-3}{-4} = \frac{3}{4}$$

$$m_{\overline{CD}} = \frac{6-3}{7-3} = \frac{3}{4}$$

$$m_{\overline{BC}} = \frac{6-6}{7-2} = 0$$

$$m_{\overline{AD}} = \frac{3-3}{-2-3} = 0$$

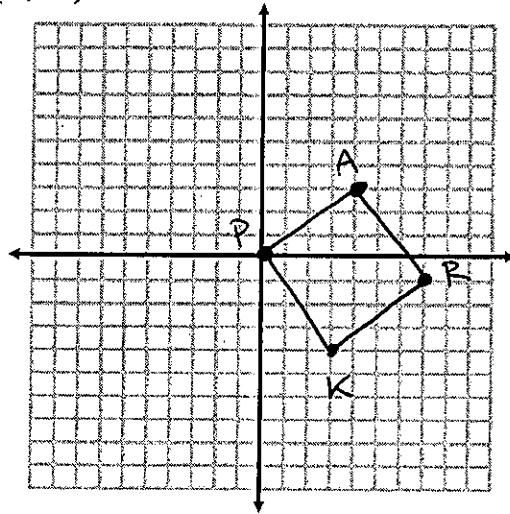


Quad. ABCD is a rhombus b/c it has 2 pairs of \parallel sides & \perp diagonals

Example 2

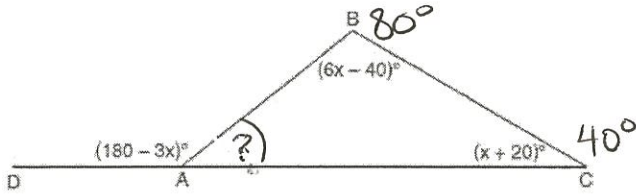
Prove that the quadrilateral with vertices
 $P(0,0)$, $A(4,3)$, $R(7,-1)$ and $K(3,-4)$
is a square

$$\begin{aligned} m_{\overline{PA}} &= \frac{3-0}{4-0} = \frac{3}{4} \\ m_{\overline{RK}} &= \frac{-1-4}{7-3} = \frac{3}{4} \quad \left. \vphantom{\begin{aligned} m_{\overline{PA}} \\ m_{\overline{RK}} \end{aligned}} \right\} \parallel \\ m_{\overline{AR}} &= \frac{3-1}{4-7} = \frac{2}{-3} \\ m_{\overline{PK}} &= \frac{0-4}{0-3} = \frac{4}{-3} \quad \left. \vphantom{\begin{aligned} m_{\overline{AR}} \\ m_{\overline{PK}} \end{aligned}} \right\} \parallel \\ m_{\overline{PR}} &= \frac{-1-0}{7-0} = -\frac{1}{7} \\ m_{\overline{AK}} &= \frac{3-4}{4-3} = -\frac{1}{1} \quad \left. \vphantom{\begin{aligned} m_{\overline{PR}} \\ m_{\overline{AK}} \end{aligned}} \right\} \perp \end{aligned}$$



Quad. PART is a square
b/c it has 2 pairs of \parallel
sides, \perp diagonals, & a rt. \angle .

- 70 In $\triangle ABC$ shown below, side \overline{AC} is extended to point D with $m\angle DAB = (180 - 3x)^\circ$, $m\angle B = (6x - 40)^\circ$, and $m\angle C = (x + 20)^\circ$.



What is $m\angle BAC$? $6x - 40 + x + 20 = 180 - 3x$

- 1) 20°
- 2) 40°
- 3) 60°
- 4) 80°

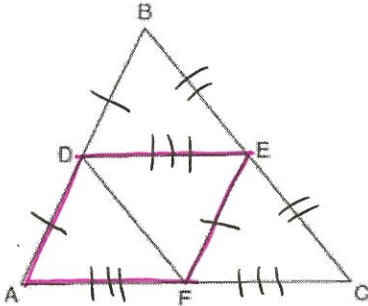
$$7x - 20 = 180 - 3x$$

$$10x = 200$$

$$x = 20$$

G.CO.C.10: MIDSEGMENTS

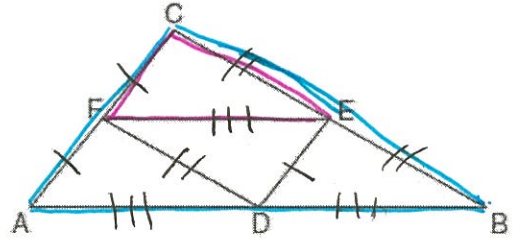
- 71 In the diagram below, \overline{DE} , \overline{DF} , and \overline{EF} are midsegments of $\triangle ABC$.



The perimeter of quadrilateral $ADEF$ is equivalent to

- 1) $AB + BC + AC$
- 2) $\frac{1}{2}AB + \frac{1}{2}AC$
- 3) $2AB + 2AC$
- 4) $AB + AC$

- 72 In the diagram below of $\triangle ABC$, D , E , and F are the midpoints of \overline{AB} , \overline{BC} , and \overline{CA} , respectively.



What is the ratio of the area of $\triangle CFE$ to the area of $\triangle CAB$?

- 1) 1:1
- 2) 1:2
- 3) 1:3
- 4) 1:4

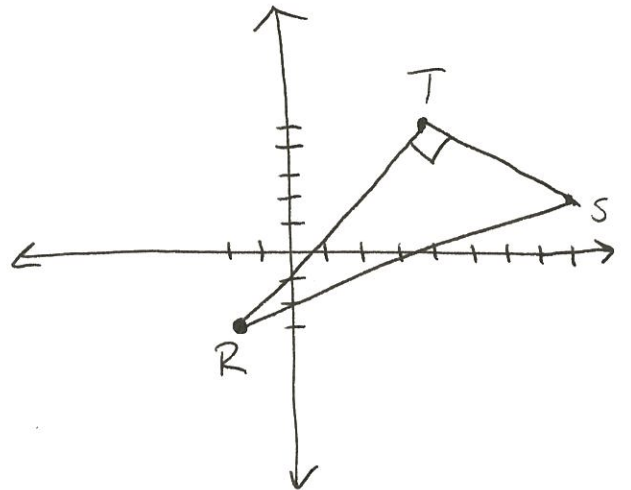
G.GPE.B.4: TRIANGLES IN THE COORDINATE PLANE

- 73 The coordinates of the vertices of $\triangle RST$ are $R(-2, -3)$, $S(8, 2)$, and $T(4, 5)$. Which type of triangle is $\triangle RST$?

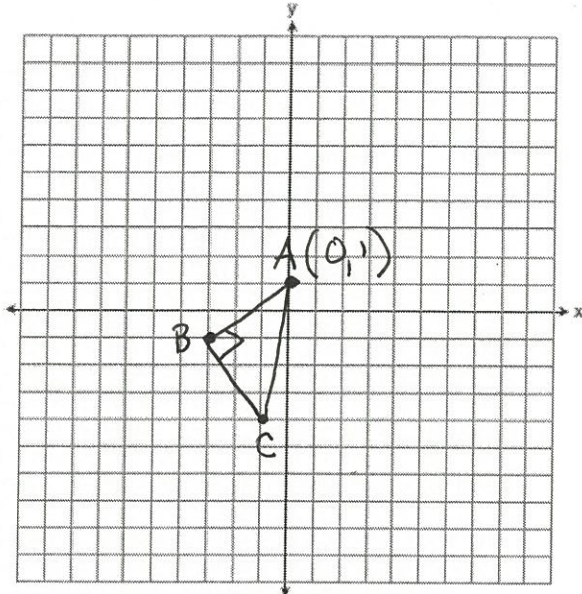
- 1) right
- 2) acute
- 3) obtuse
- 4) equiangular

$$m_{\overline{ST}} = \frac{5-2}{4-8} = \frac{3}{-4}$$

$$m_{\overline{RT}} = \frac{-3-5}{-2-4} = \frac{4}{3}$$



- 74 Triangle ABC has vertices with $A(x, 3)$, $B(-3, -1)$, and $C(-1, -4)$. Determine and state a value of x that would make triangle ABC a right triangle. Justify why $\triangle ABC$ is a right triangle. [The use of the set of axes below is optional.]



$$m_{\overline{BC}} = \frac{-1 - (-4)}{-3 - (-1)} = \frac{3}{-2}$$

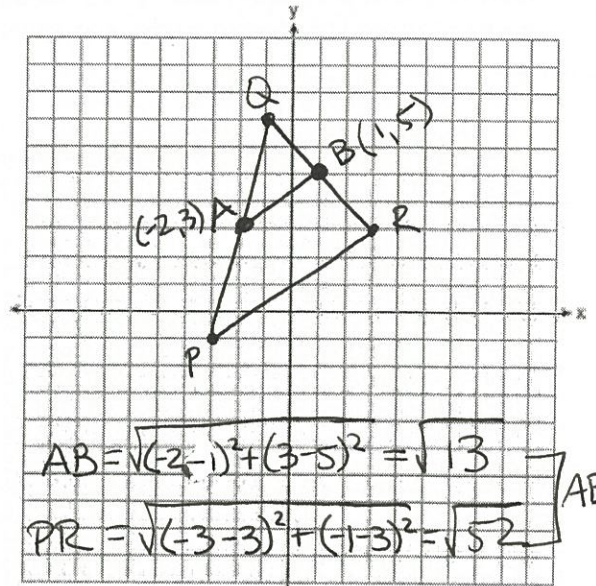
$$\text{slope } \overline{AB} = \frac{2}{3} \quad \left. \begin{array}{l} \\ \end{array} \right\} (-) \text{ rec.}$$

$$\boxed{x = 0}$$

$$m_{\overline{AB}} = \frac{3 - (-1)}{0 - (-3)} = \frac{2}{3}$$

$\triangle ABC$ is a right \triangle
 b/c it has a right \angle at A .
 The slopes of \overline{BC} & \overline{AB} are
 neg rec., so they are \perp .

- 75 Triangle PQR has vertices $P(-3, -1)$, $Q(-1, 7)$, and $R(3, 3)$, and points A and B are midpoints of \overline{PQ} and \overline{RQ} , respectively. Use coordinate geometry to prove that \overline{AB} is parallel to \overline{PR} and is half the length of \overline{PR} . [The use of the set of axes below is optional.]



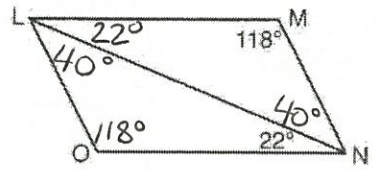
$$m_{\overline{AB}} = \frac{3 - 5}{-2 - 1} = \frac{-2}{-3} = \frac{2}{3}$$

$$m_{\overline{PR}} = \frac{-1 - 3}{-3 - 3} = \frac{-4}{-6} = \frac{2}{3}$$

parallel lines have the same slope

POLYGONS
G.CO.C.11: INTERIOR AND EXTERIOR ANGLES OF POLYGONS

- 76 The diagram below shows parallelogram $LMNO$ with diagonal \overline{LN} , $m\angle M = 118^\circ$, and $m\angle LNO = 22^\circ$.

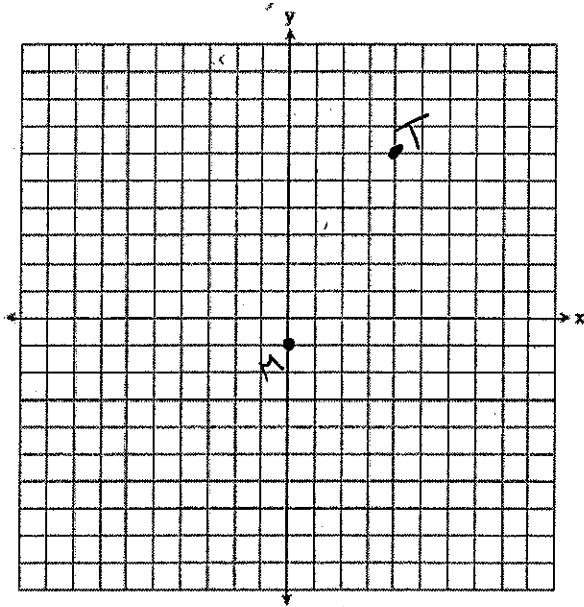


Explain why $m\angle NLO$ is 40 degrees.
 $\angle LON$ is 118° b/c opp. \angle 's of a parallelogram are \cong . A \triangle 's \angle measures add up to 180° :
 $118 + 22 = 140$ so $\angle NLO$ must be 40 degrees. 14

G.GPE.B.4: QUADRILATERALS IN THE COORDINATE PLANE

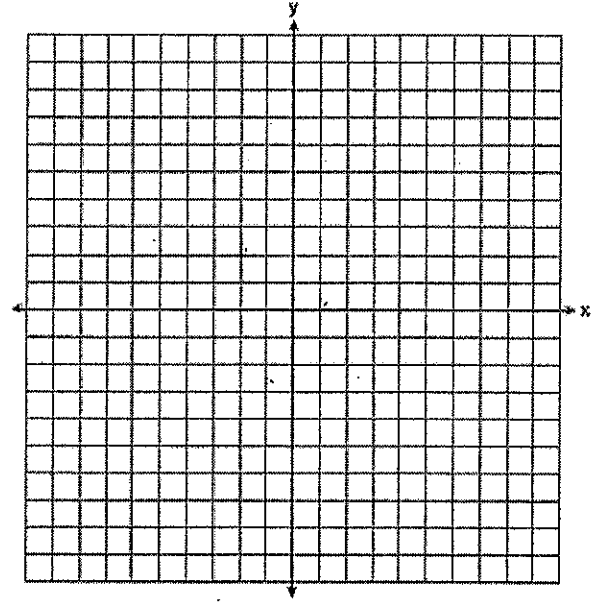
- 93 In rhombus $MATH$, the coordinates of the endpoints of the diagonal \overline{MT} are $M(0, -1)$ and $T(4, 6)$. Write an equation of the line that contains diagonal \overline{AH} . [Use of the set of axes below is optional.] Using the given information, explain how you know that your line contains diagonal \overline{AH} .

* sep. sheet



- 95 In the coordinate plane, the vertices of $\triangle RST$ are $R(6, -1)$, $S(1, -4)$, and $T(-5, 6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]

* sep. sheet



- 94 A quadrilateral has vertices with coordinates $(-3, 1)$, $(0, 3)$, $(5, 2)$, and $(-1, -2)$. Which type of quadrilateral is this?
- 1) rhombus
 - 2) rectangle
 - 3) square
 - 4) trapezoid

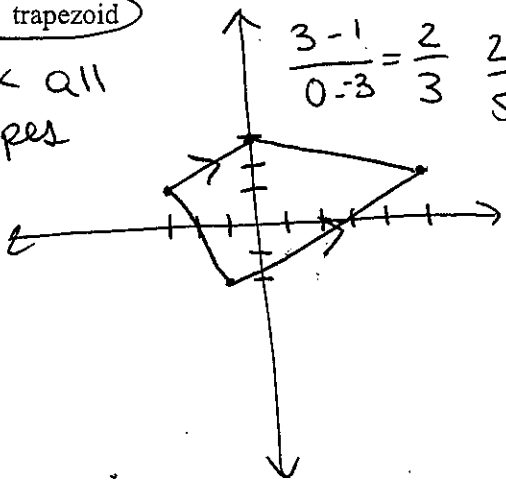
*check all slopes

$$\frac{-2-1}{-1-3} = \frac{-3}{-4} = \frac{3}{4}$$

$$\frac{3-2}{0-5} = \frac{1}{-5} = -\frac{1}{5}$$

$$\frac{3-1}{0-3} = \frac{2}{-3} = -\frac{2}{3}$$

$$\frac{2-2}{5-1} = \frac{0}{4} = 0$$



- 96 The diagonals of rhombus $TEAM$ intersect at $P(2, 1)$. If the equation of the line that contains diagonal \overline{TA} is $y = -x + 3$, what is the equation of a line that contains diagonal \overline{EM} ?

- 1) $y = x - 1$
- 2) $y = x - 3$
- 3) $y = -x - 1$
- 4) $y = -x - 3$

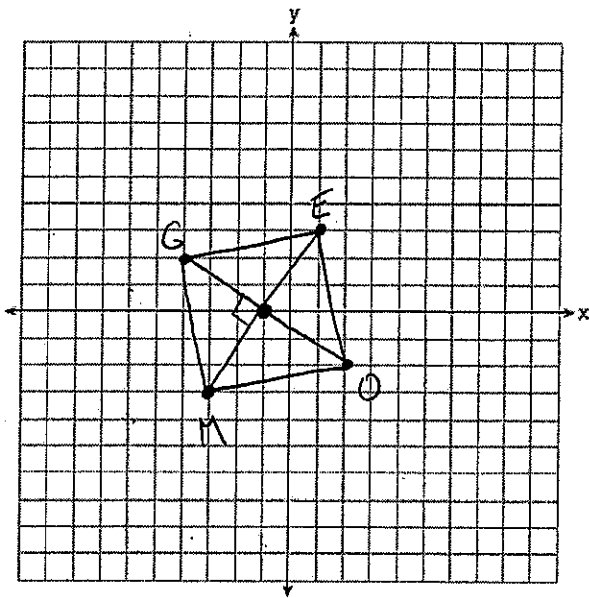
$P(2, 1)$
 $m = 1$
 $y - 1 = 1(x - 2)$
 $y = x - 1$

97 Parallelogram $ABCD$ has coordinates $A(0,7)$ and $C(2,1)$. Which statement would prove that $ABCD$ is a rhombus?

- 1) The midpoint of \overline{AC} is $(1,4)$. $m_{\overline{AC}} = \frac{7-1}{0-2} = \frac{6}{-2} = -3$
- 2) The length of \overline{BD} is $\sqrt{40}$.
- 3) The slope of \overline{BD} is $\frac{1}{3}$.
- 4) The slope of \overline{AB} is $\frac{1}{3}$.

*rhombus has
⊥ diagonals

98 In square $GEOM$, the coordinates of G are $(2,-2)$ and the coordinates of O are $(-4,2)$. Determine and state the coordinates of vertices E and M . [The use of the set of axes below is optional.]



*diagonals of square
are ⊥ bisectors

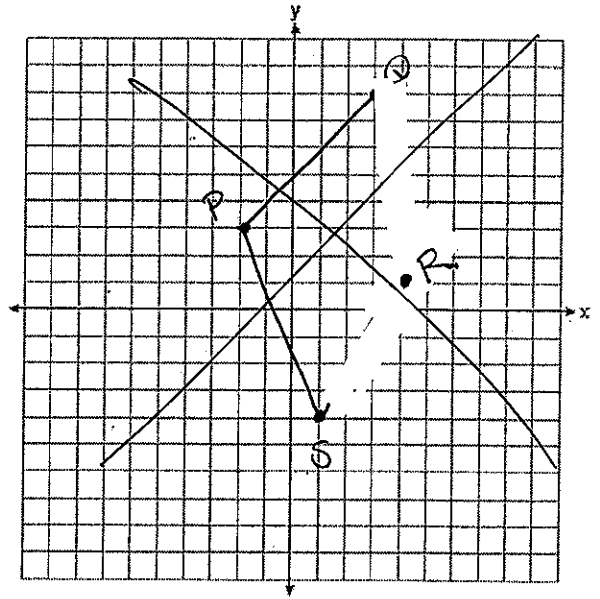
$$m_{\overline{GO}} = \frac{-2-2}{2-(-4)} = \frac{-4}{6} = -\frac{2}{3} \rightarrow m_{\overline{EM}} = \frac{3}{2}$$

$$\text{midpt } \overline{GO} = \left(\frac{2+(-4)}{2}, \frac{-2+2}{2} \right) = (-1, 0)$$

$$E(1, 3) \text{ \& } M(-3, -3)$$

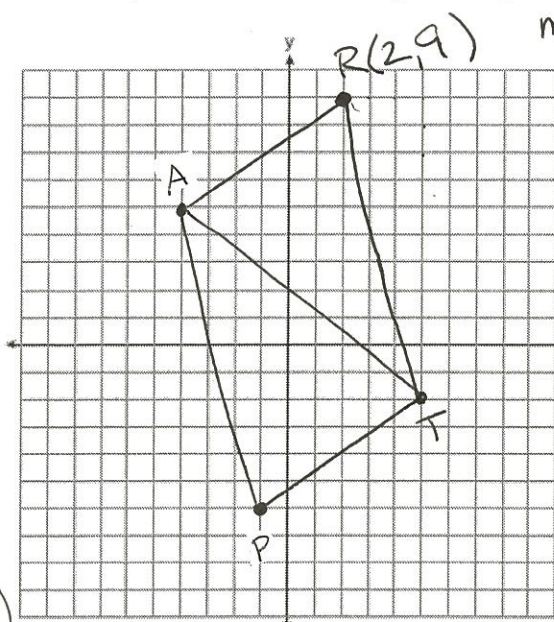
24

99 Quadrilateral $PQRS$ has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that $PQRS$ is a rhombus. Prove that $PQRS$ is *not* a square. [The use of the set of axes below is optional.]



* 99 sep. sheet

- 100 In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram.



$m\overline{AP} = -\frac{11}{3}$
 $m\overline{RT} = -\frac{11}{3} \parallel$
 $m\overline{AR} = \frac{2}{3}$
 $m\overline{PT} = \frac{2}{3} \parallel$
 Quad. $PART$ is a parallelogram b/c opp sides are \parallel .

$\triangle PAT$ is
 sos. b/c
 + has
 $2 \cong$
 sides

$PA = \sqrt{(-1-4)^2 + (-6-5)^2} = \sqrt{130}$
 $AT = \sqrt{(-4-5)^2 + (5-2)^2} = \sqrt{130}$

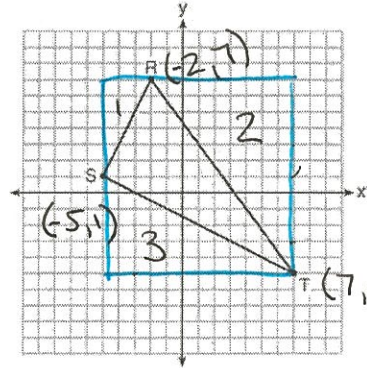
G.GPE.B.7: POLYGONS IN THE COORDINATE PLANE

- 101 The endpoints of one side of a regular pentagon are $(-1,4)$ and $(2,3)$. What is the perimeter of the pentagon?

1) $\sqrt{10}$
 2) $5\sqrt{10}$
 3) $5\sqrt{2}$
 4) $25\sqrt{2}$

$d = \sqrt{(-1-2)^2 + (4-3)^2}$
 $= \sqrt{(-3)^2 + (1)^2}$
 $= \sqrt{9+1}$
 $= \sqrt{10}$
 $5 \times \sqrt{10} = 5\sqrt{10}$

- 102 Triangle RST is graphed on the set of axes below.



$144 - 99 = 45$

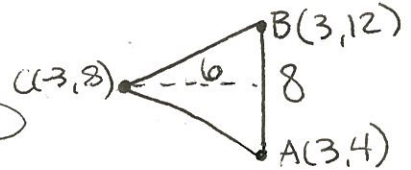
How many square units are in the area of $\triangle RST$?

- 1) $9\sqrt{3} + 15$
 2) $9\sqrt{5} + 15$
 3) 45
 4) 90

$A\Delta 1 = \frac{1}{2}(3)(6) = 9$
 $A\Delta 2 = \frac{1}{2}(9)(12) = 54$
 $A\Delta 3 = \frac{1}{2}(6)(12) = 36$
 $A_{\text{rec}} = (12)(12) = 144$

- 103 The coordinates of vertices A and B of $\triangle ABC$ are $A(3,4)$ and $B(3,12)$. If the area of $\triangle ABC$ is 24 square units, what could be the coordinates of point C ?

- 1) $(3,6)$
 2) $(8,-3)$
 3) $(-3,8)$
 4) $(6,3)$



$\frac{1}{2}(8)b = 24$
 $b = 6$

- 104 The vertices of square $RSTV$ have coordinates $R(-1,5)$, $S(-3,1)$, $T(-7,3)$, and $V(-5,7)$. What is the perimeter of $RSTV$?

- 1) $\sqrt{20}$
 2) $\sqrt{40}$
 3) $4\sqrt{20}$
 4) $4\sqrt{40}$

$RS = \sqrt{(-1+3)^2 + (5-1)^2}$
 $= \sqrt{2^2 + 4^2}$
 $= \sqrt{4+16}$
 $= \sqrt{20}$

$4 \times \sqrt{20} = 4\sqrt{20}$

Formulas

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Keywords:

- Parallel
- Perpendicular
- Altitude

Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Keywords:

- Congruent
- Length
- Isosceles

Midpoint

$$mdpt = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Keywords:

- Median
- Bisect
- Point of intersection of diagonals
- Center of a circle

- Parallel lines have EQUAL slopes.

- Perpendicular lines have NEGATIVE RECIPROCAL slopes.

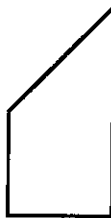
- Congruent segments have EQUAL distances.

- Segments that bisect each other have the SAME midpoints.

Trapezoid

- 2 slopes

___ is a trapezoid because it has one pair of parallel sides.



Parallelogram

- 4 slopes

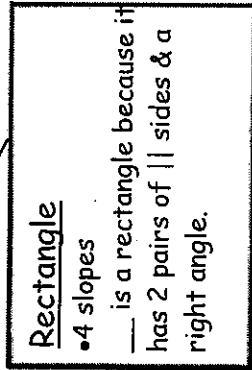
___ is a parallelogram because it has 2 pairs of parallel sides.



Rectangle

- 4 slopes

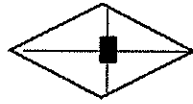
___ is a rectangle because it has 2 pairs of || sides & a right angle.



Rhombus

- 6 slopes

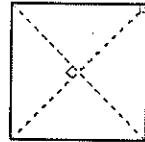
___ is a rhombus because it has 2 pairs of || sides and ⊥ diagonals.



Square

- 6 slopes

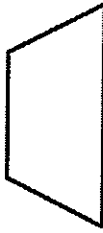
___ is a square because it has 2 pairs of || sides, ⊥ diagonals and a right angle.



Isosceles Trapezoid

- 2 slopes, 2 distance

___ is an isosceles trapezoid because it has 1 pair of || sides and non-|| sides are congruent.



Isosceles Triangle

- 2 distances

___ is an isosceles triangle because it has 2 ≅ sides

Right Triangle

- 2 slopes

___ is a right triangle because it has a right angle.

Isosceles Right Triangle

- 2 slopes
- 2 distances

___ is an isosceles right triangle because it has 2 congruent sides and a right angle.

