



# Geometry

## Unit 4-12

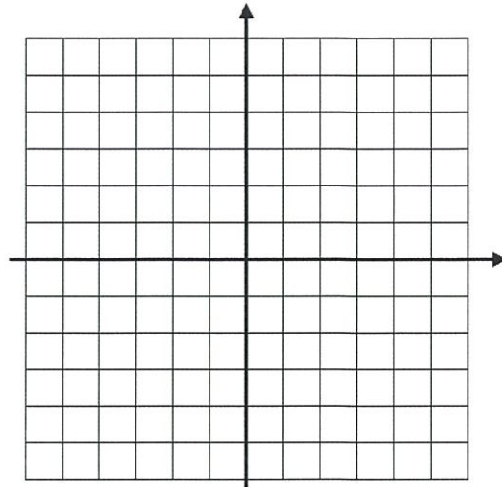
### Coordinate Geometry Proofs

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## Lesson 1 Triangle Proofs

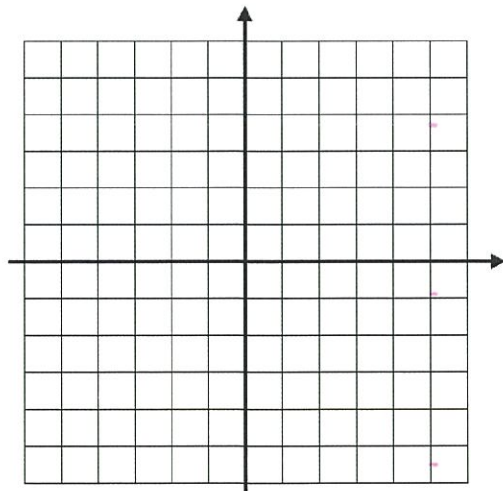
### Example 1:

The vertices of  $\triangle ABC$  are  $A(-2, 4)$ ,  $B(-2, 8)$  and  $C(-5, 6)$ .  
Prove  $\triangle ABC$  is isosceles.



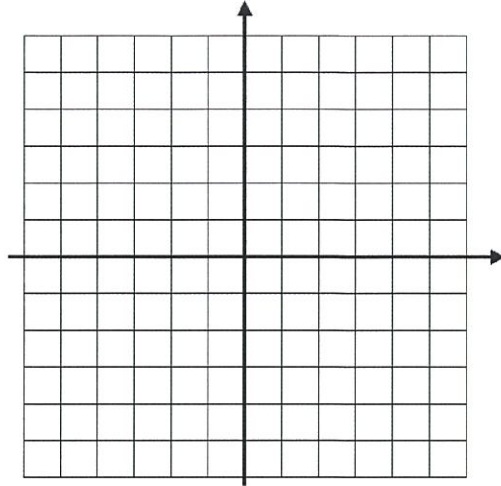
### Example 2:

The vertices of  $\triangle JEN$  are  $J(-4, 1)$ ,  $E(-2, -3)$  and  $N(2, -1)$ .  
Prove  $\triangle JEN$  is a right triangle.



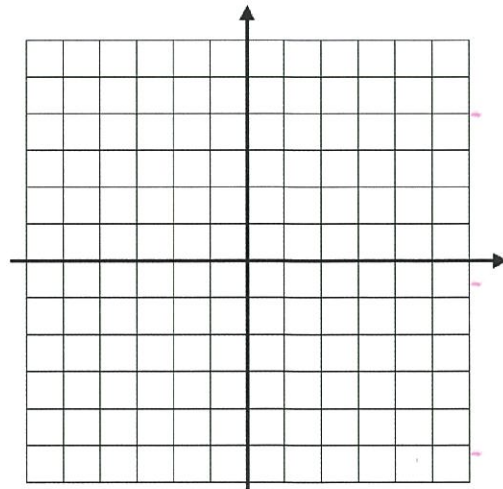
### Example 3:

Prove that  $A(4,-1)$ ,  $B(5,6)$ ,  $C(1,3)$  is an isosceles right triangle.



### Example 4:

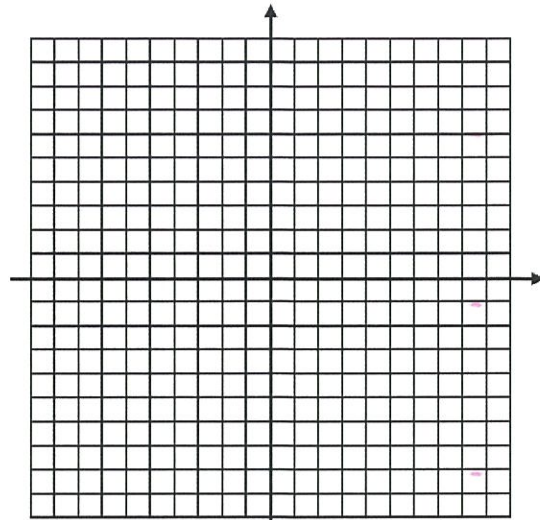
The coordinates of  $\triangle ABC$  are  $A(0,0)$ ,  $B(2,6)$ , and  $C(4,2)$ . Using coordinate geometry, prove that if the midpoints of sides  $\overline{AB}$  and  $\overline{AC}$  are joined, the segment formed is parallel to the third side and equal to one-half the length of the third side.



## Example 5:

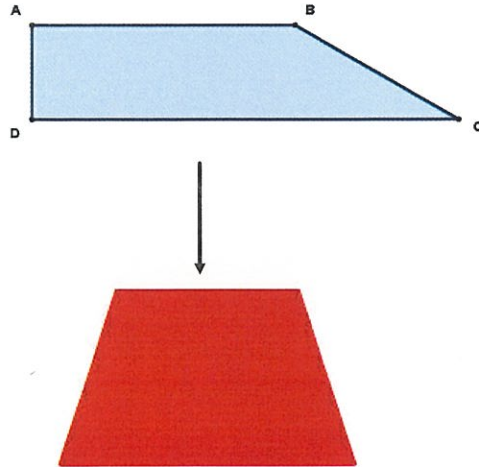
The vertices of  $\triangle NYS$  are  $N(-2,-1)$ ,  $Y(0,10)$ , and  $S(10,5)$ . The coordinates of point  $T$  are  $(4,2)$ .

- (a) Prove that  $\overline{YT}$  is a median.
- (b) Prove that  $\overline{YT}$  is an altitude.
- (c) Find the area of  $\triangle NYS$ .



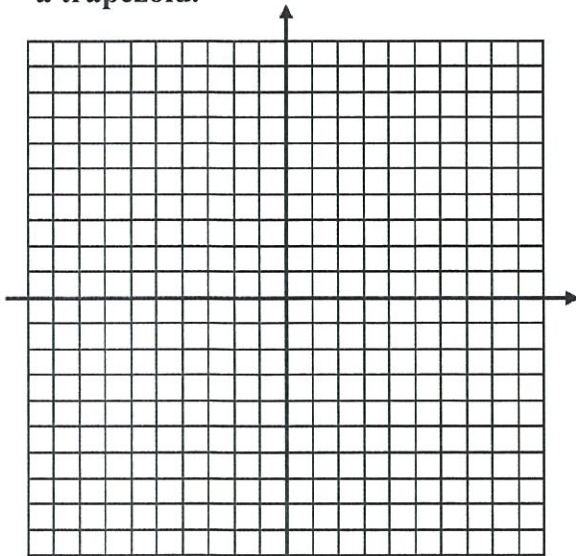
## Unit 12 Lesson 2

### Trapezoid and Isosceles Trapezoid



### Example 1

Prove that quadrilateral MILK with the vertices  $M(-2,3)$ ,  $I(3, 1)$ ,  $L(7, 9)$ , and  $K(1, 9)$  is a trapezoid.





### Example 2

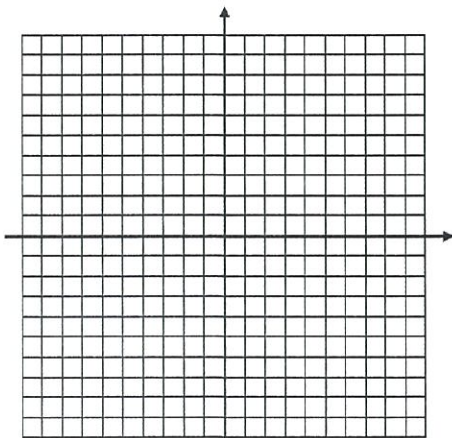
Quadrilateral JAKE has coordinates  
 $J(0, 3a)$ ,  $A(3a, 3a)$ ,  $K(4a, 0)$  and  $E(-a, 0)$ .

Prove by coordinate geometry that  
quadrilateral JAKE is an isosceles trapezoid.

### Example 3

Quadrilateral BOAT has coordinates  
 $B(2, 1)$ ,  $O(6, 3)$ ,  $A(8, 7)$  and  $T(4, 5)$ .

Prove by coordinate geometry that the diagonals of BOAT bisect each other.

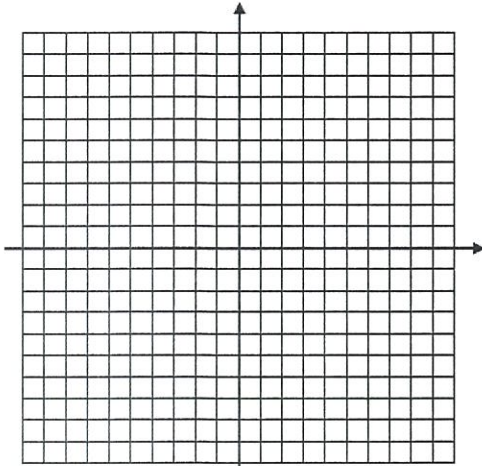


### Example 4

Given quadrilateral TRAP with coordinates  $T(-5, 7)$ ,  $R(-3, -4)$  and  $A(9, 5)$ .

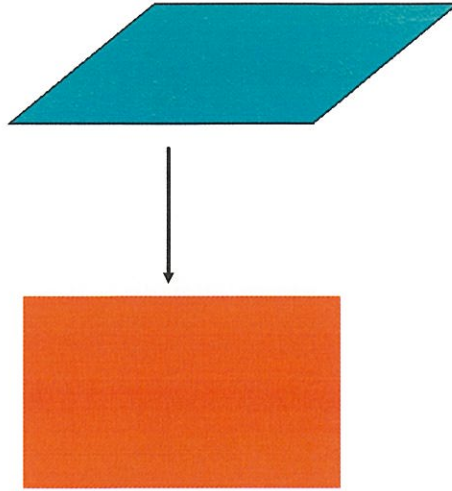
Determine and state coordinates of  $P$  that would make TRAP a trapezoid.

Then prove, using coordinate geometry that TRAP is a trapezoid.

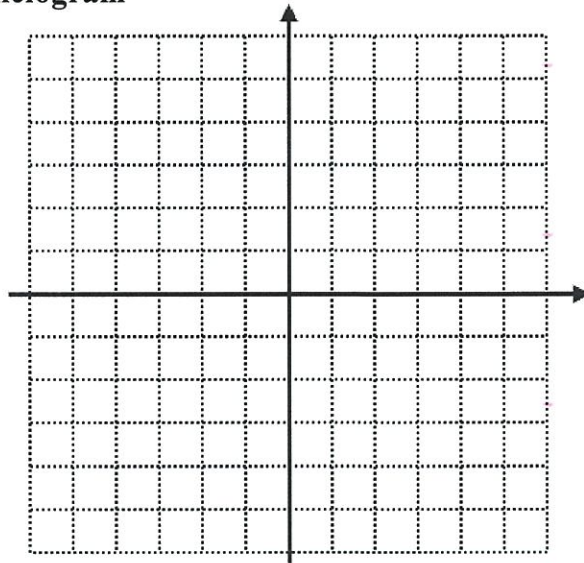


## Unit 12 Lesson 3

# Parallelogram and Rectangle



**Example 1** Prove that the quadrilateral with the coordinates  $L(-2,3)$ ,  $M(4,3)$ ,  $N(2,-2)$  and  $O(-4,-2)$  is a parallelogram



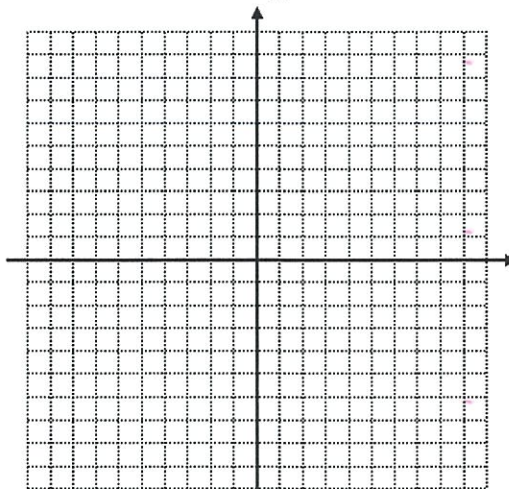


## Example 2

Prove that the quadrilateral with the coordinates  $M(0, 0)$ ,  $A(r, t)$ ,  $T(s, t)$  and  $H(s - r, 0)$  is a parallelogram.

## Example 3

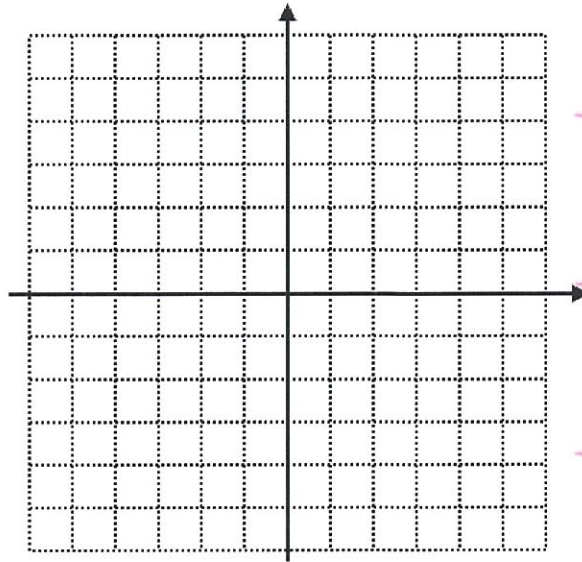
Quadrilateral ABCD has vertices  $A(6, 0)$ ,  $B(3, 9)$ ,  $C(-3, 7)$  and  $D(0, -2)$ .  
Prove that ABCD is a rectangle.



### Example 4

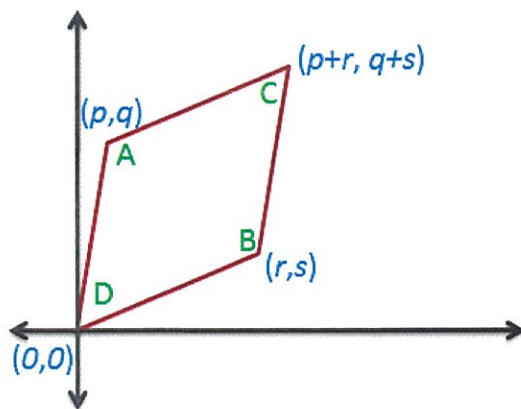
Prove that quadrilateral RATS is a rectangle.

R(-5, -3) A(-5, 1) T(1, 1) S(1, -3)



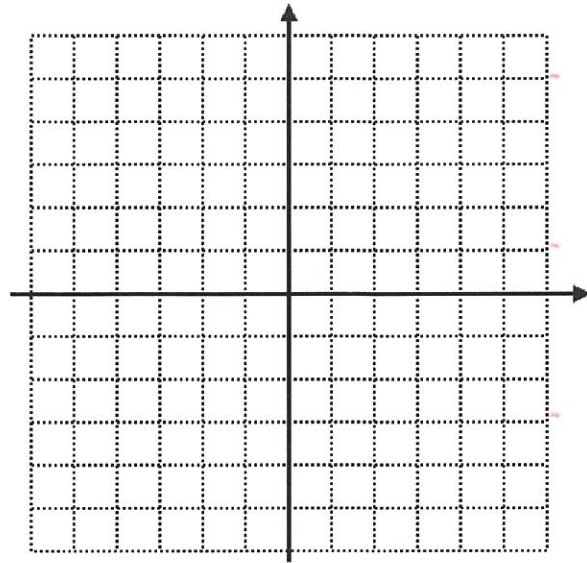
### Example 5

Is quadrilateral ABCD a rectangle? Prove it.



### Example 6

Prove that the quadrilateral with coordinates  $R(0,5)$ ,  $S(3,4)$ ,  $T(0,-5)$  and  $U(-3,-4)$  is a parallelogram.

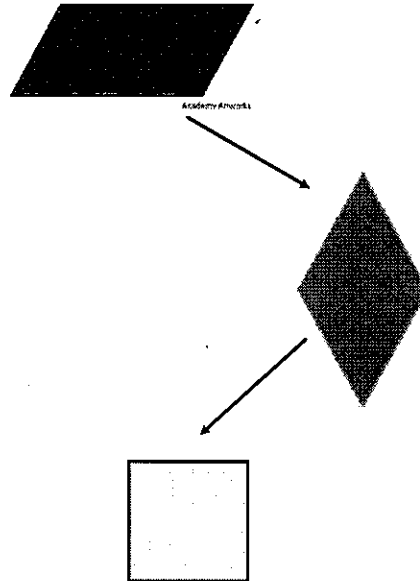


### Example 7

Prove that the quadrilateral with the coordinates  $R(0, 0)$ ,  $S(r, s)$ ,  $T(r, s + t)$  and  $U(0, t)$  is a parallelogram.

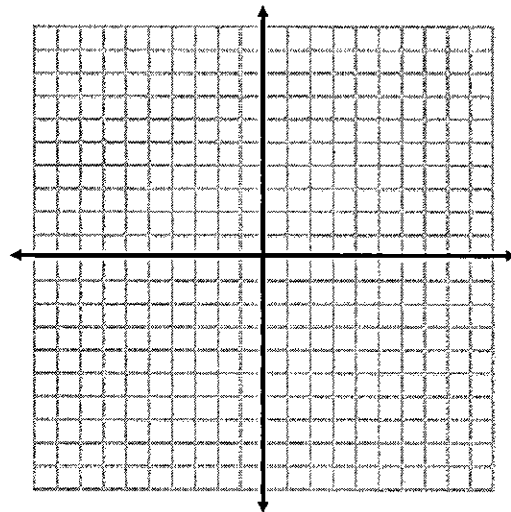
# Unit 12 Lesson 4

## Rhombus and Square



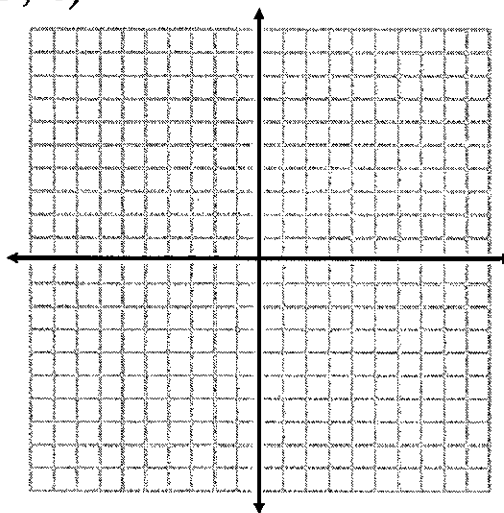
### Example 1

Prove that a quadrilateral with the vertices  $A(-2,3)$ ,  $B(2,6)$ ,  $C(7,6)$  and  $D(3,3)$  is a rhombus.

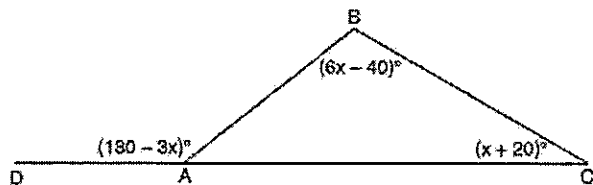


## Example 2

Prove that the quadrilateral with vertices  $P(0,0)$ ,  $A(4,3)$ ,  $R(7,-1)$  and  $K(3,-4)$  is a square



- 70 In  $\triangle ABC$  shown below, side  $\overline{AC}$  is extended to point  $D$  with  $m\angle DAB = (180 - 3x)^\circ$ ,  $m\angle B = (6x - 40)^\circ$ , and  $m\angle C = (x + 20)^\circ$ .

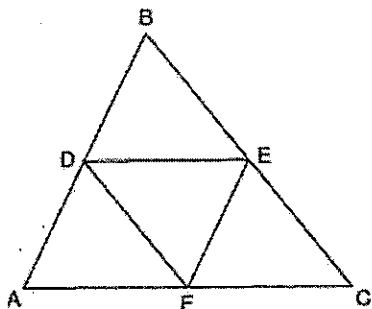


What is  $m\angle BAC$ ?

- 1)  $20^\circ$
- 2)  $40^\circ$
- 3)  $60^\circ$
- 4)  $80^\circ$

G.CO.C.10: MIDSEGMENTS

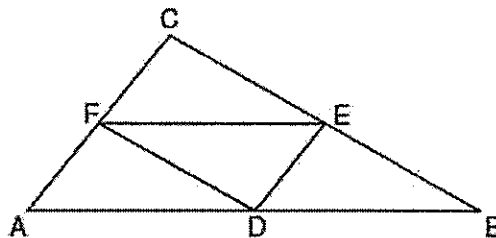
- 71 In the diagram below,  $\overline{DE}$ ,  $\overline{DF}$ , and  $\overline{EF}$  are midsegments of  $\triangle ABC$ .



The perimeter of quadrilateral  $ADEF$  is equivalent to

- 1)  $AB + BC + AC$
- 2)  $\frac{1}{2}AB + \frac{1}{2}AC$
- 3)  $2AB + 2AC$
- 4)  $AB + AC$

- 72 In the diagram below of  $\triangle ABC$ ,  $D$ ,  $E$ , and  $F$  are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{CA}$ , respectively.



What is the ratio of the area of  $\triangle CFE$  to the area of  $\triangle CAB$ ?

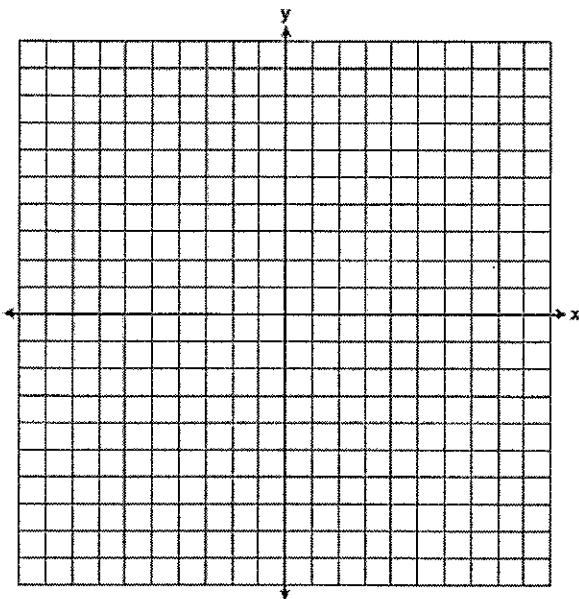
- 1) 1:1
- 2) 1:2
- 3) 1:3
- 4) 1:4

G.GPE.B.4: TRIANGLES IN THE COORDINATE PLANE

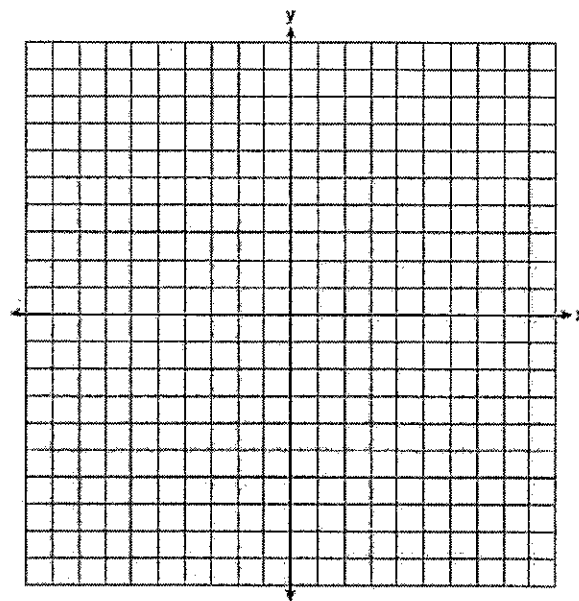
- 73 The coordinates of the vertices of  $\triangle RST$  are  $R(-2, -3)$ ,  $S(8, 2)$ , and  $T(4, 5)$ . Which type of triangle is  $\triangle RST$ ?
- 1) right
  - 2) acute
  - 3) obtuse
  - 4) equiangular



- 74 Triangle  $ABC$  has vertices with  $A(x,3)$ ,  $B(-3,-1)$ , and  $C(-1,-4)$ . Determine and state a value of  $x$  that would make triangle  $ABC$  a right triangle. Justify why  $\triangle ABC$  is a right triangle. [The use of the set of axes below is optional.]



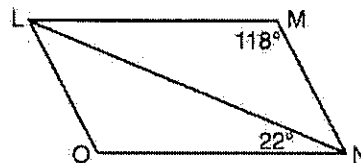
- 75 Triangle  $PQR$  has vertices  $P(-3,-1)$ ,  $Q(-1,7)$ , and  $R(3,3)$ , and points  $A$  and  $B$  are midpoints of  $\overline{PQ}$  and  $\overline{RQ}$ , respectively. Use coordinate geometry to prove that  $\overline{AB}$  is parallel to  $\overline{PR}$  and is half the length of  $\overline{PR}$ . [The use of the set of axes below is optional.]



## POLYGONS

### G.CO.C.11: INTERIOR AND EXTERIOR ANGLES OF POLYGONS

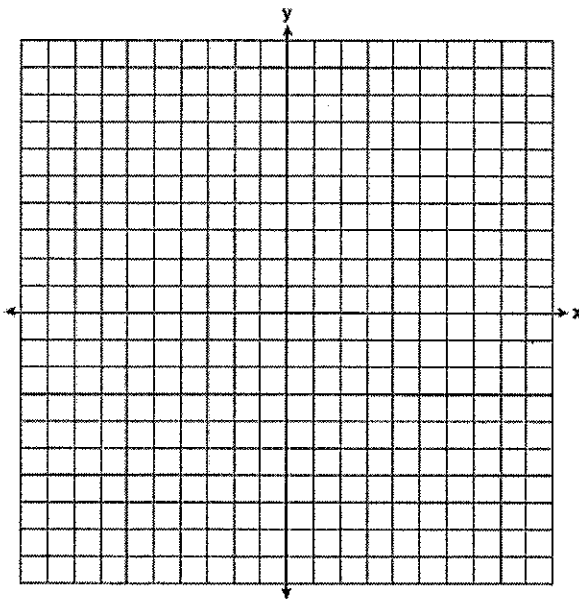
- 76 The diagram below shows parallelogram  $LMNO$  with diagonal  $\overline{LN}$ ,  $m\angle M = 118^\circ$ , and  $m\angle LNO = 22^\circ$ .



Explain why  $m\angle NLO$  is 40 degrees.

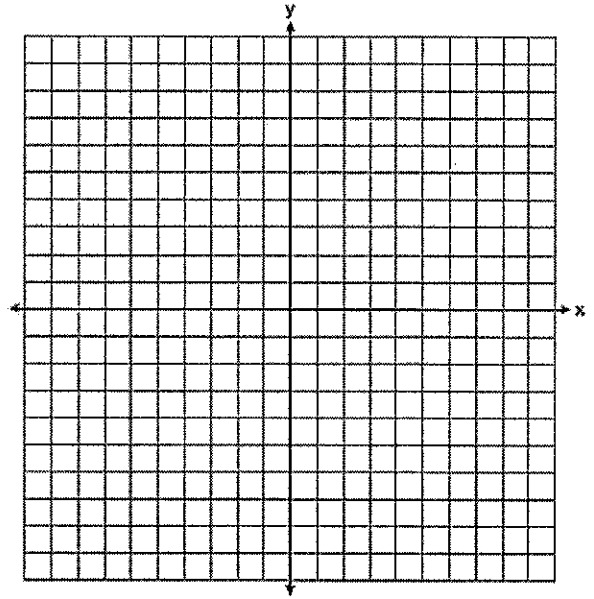
G.GPE.B.4: QUADRILATERALS IN THE COORDINATE PLANE

- 93 In rhombus  $MATH$ , the coordinates of the endpoints of the diagonal  $\overline{MT}$  are  $M(0,-1)$  and  $T(4,6)$ . Write an equation of the line that contains diagonal  $\overline{AH}$ . [Use of the set of axes below is optional.] Using the given information, explain how you know that your line contains diagonal  $\overline{AH}$ .



- 94 A quadrilateral has vertices with coordinates  $(-3, 1)$ ,  $(0, 3)$ ,  $(5, 2)$ , and  $(-1, -2)$ . Which type of quadrilateral is this?
- 1) rhombus
  - 2) rectangle
  - 3) square
  - 4) trapezoid

- 95 In the coordinate plane, the vertices of  $\triangle RST$  are  $R(6, -1)$ ,  $S(1, -4)$ , and  $T(-5, 6)$ . Prove that  $\triangle RST$  is a right triangle. State the coordinates of point  $P$  such that quadrilateral  $RSTP$  is a rectangle. Prove that your quadrilateral  $RSTP$  is a rectangle. [The use of the set of axes below is optional.]

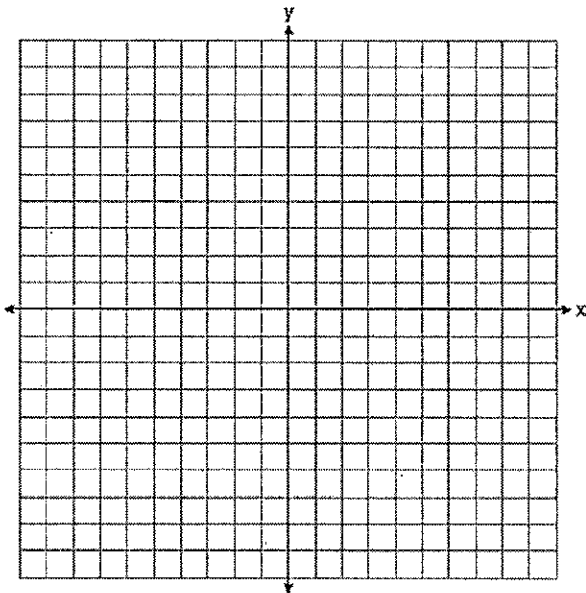


- 96 The diagonals of rhombus  $TEAM$  intersect at  $P(2, 1)$ . If the equation of the line that contains diagonal  $\overline{TA}$  is  $y = -x + 3$ , what is the equation of a line that contains diagonal  $\overline{EM}$ ?
- 1)  $y = x - 1$
  - 2)  $y = x - 3$
  - 3)  $y = -x - 1$
  - 4)  $y = -x - 3$

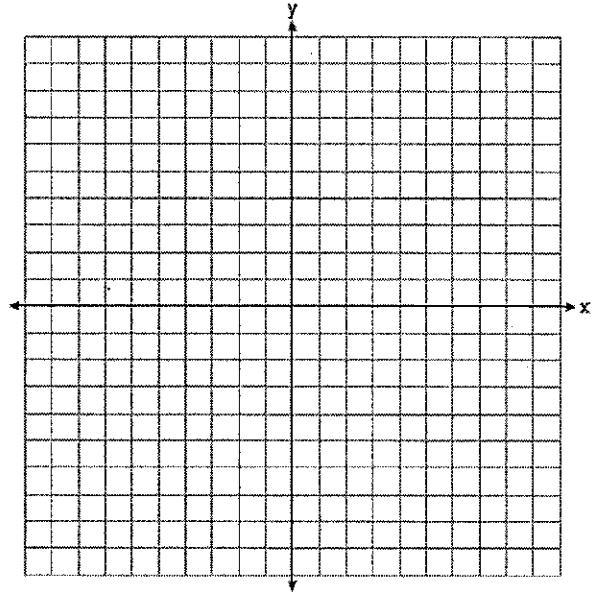
97 Parallelogram  $ABCD$  has coordinates  $A(0,7)$  and  $C(2,1)$ . Which statement would prove that  $ABCD$  is a rhombus?

- 1) The midpoint of  $\overline{AC}$  is  $(1,4)$ .
- 2) The length of  $\overline{BD}$  is  $\sqrt{40}$ .
- 3) The slope of  $\overline{BD}$  is  $\frac{1}{3}$ .
- 4) The slope of  $\overline{AB}$  is  $\frac{1}{3}$ .

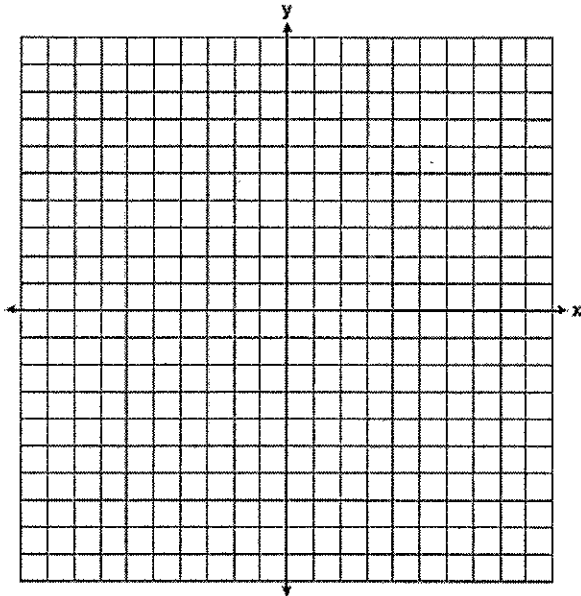
98 In square  $GEOM$ , the coordinates of  $G$  are  $(2,-2)$  and the coordinates of  $O$  are  $(-4,2)$ . Determine and state the coordinates of vertices  $E$  and  $M$ . [The use of the set of axes below is optional.]



99 Quadrilateral  $PQRS$  has vertices  $P(-2,3)$ ,  $Q(3,8)$ ,  $R(4,1)$ , and  $S(-1,-4)$ . Prove that  $PQRS$  is a rhombus. Prove that  $PQRS$  is *not* a square. [The use of the set of axes below is optional.]



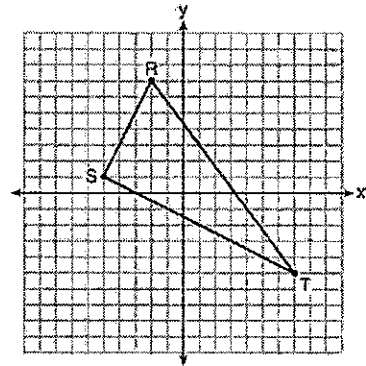
- 100 In the coordinate plane, the vertices of triangle  $PAT$  are  $P(-1, -6)$ ,  $A(-4, 5)$ , and  $T(5, -2)$ . Prove that  $\triangle PAT$  is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of  $R$  so that quadrilateral  $PART$  is a parallelogram. Prove that quadrilateral  $PART$  is a parallelogram.



G.GPE.B.7: POLYGONS IN THE COORDINATE PLANE

- 101 The endpoints of one side of a regular pentagon are  $(-1, 4)$  and  $(2, 3)$ . What is the perimeter of the pentagon?
- 1)  $\sqrt{10}$
  - 2)  $5\sqrt{10}$
  - 3)  $5\sqrt{2}$
  - 4)  $25\sqrt{2}$

- 102 Triangle  $RST$  is graphed on the set of axes below.



How many square units are in the area of  $\triangle RST$ ?

- 1)  $9\sqrt{3} + 15$
  - 2)  $9\sqrt{5} + 15$
  - 3) 45
  - 4) 90
- 103 The coordinates of vertices  $A$  and  $B$  of  $\triangle ABC$  are  $A(3, 4)$  and  $B(3, 12)$ . If the area of  $\triangle ABC$  is 24 square units, what could be the coordinates of point  $C$ ?
- 1)  $(3, 6)$
  - 2)  $(8, -3)$
  - 3)  $(-3, 8)$
  - 4)  $(6, 3)$
- 104 The vertices of square  $RSTV$  have coordinates  $R(-1, 5)$ ,  $S(-3, 1)$ ,  $T(-7, 3)$ , and  $V(-5, 7)$ . What is the perimeter of  $RSTV$ ?
- 1)  $\sqrt{20}$
  - 2)  $\sqrt{40}$
  - 3)  $4\sqrt{20}$
  - 4)  $4\sqrt{40}$

Formulas

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Keywords:

- Parallel
- Perpendicular
- Altitude

Distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Keywords:

- Congruent
- Length
- Isosceles

Midpoint

$$mdpt = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Keywords:

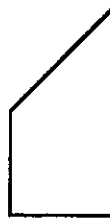
- Median
- Bisect
- Point of intersection of diagonals
- Center of a circle

- Parallel lines have EQUAL slopes.
- Perpendicular lines have NEGATIVE RECIPROCAL slopes.
- Congruent segments have EQUAL distances.
- Segments that bisect each other have the SAME midpoints.

Trapezoid

- 2 slopes

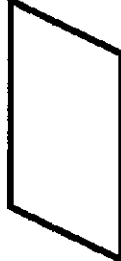
\_\_\_ is a trapezoid because it has one pair of parallel sides.



Parallelogram

- 4 slopes

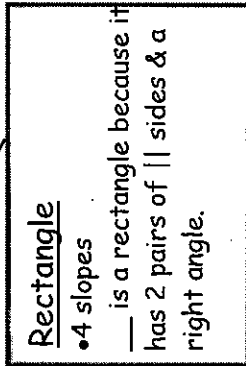
\_\_\_ is a parallelogram because it has 2 pairs of parallel sides.



Rectangle

- 4 slopes

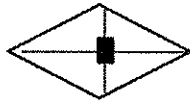
\_\_\_ is a rectangle because it has 2 pairs of || sides & a right angle.



Rhombus

- 6 slopes

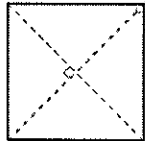
\_\_\_ is a rhombus because it has 2 pairs of || sides and ⊥ diagonals.



Square

- 6 slopes

\_\_\_ is a square because it has 2 pairs of || sides, ⊥ diagonals and a right angle.



Isosceles Trapezoid

- 2 slopes, 2 distance

\_\_\_ is an isosceles trapezoid because it has 1 pair of || sides and non-|| sides are congruent.



Isosceles Triangle

- 2 distances

\_\_\_ is an isosceles triangle because it has 2 ≅ sides

Right Triangle

- 2 slopes

\_\_\_ is a right triangle because it has a right angle.

Isosceles Right Triangle

- 2 slopes
- 2 distances

\_\_\_ is an isosceles right triangle because it has 2 congruent sides and a right angle.