Geometry

Unit 2-7

Dilations

Review: Proportional Relationships, Ratios and Scale Factors

x	y
0	0
1	3
2	6
3	9
4	12

[illegible]	[illegible]
2	8
4	16
6	24

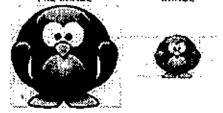
x	y
2	3
4	6
8	12

Unit 7: Dilations and Similarity

Lesson 1 Intro to Dilations

PRE-IMAGE IMAGE

PRE-IMAGE IMAGE



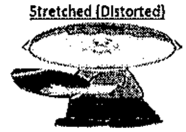
Original

Dilated (Proportional)

Dilated (Proportional)

Original

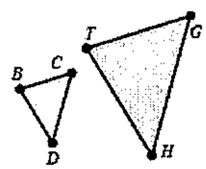
Stretched (Distorted)

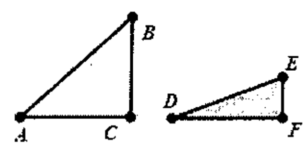


Stretched (Distorted)

Determine whether the following are stretch or dilation transformations.

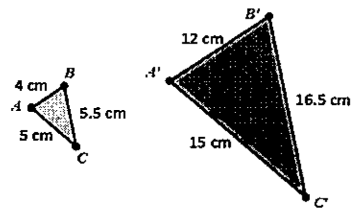
$R(x, y) \longrightarrow (x, 3y)$



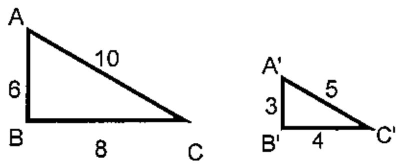


$W(x, y) \longrightarrow (\sqrt{5}x, \sqrt{5}y)$

The length of each side of the image is equal to the length of the corresponding side of the pre-image multiplied by the *scale factor*.



What is the scale factor of the dilation that maps ΔABC onto ΔA'B'C'?

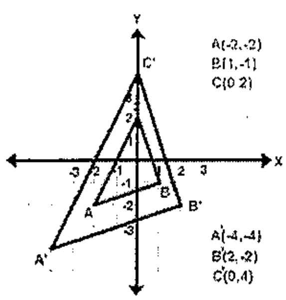


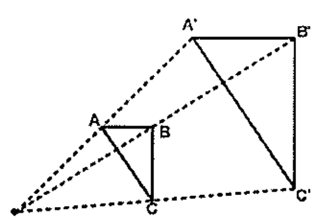
What is the scale factor of the dilation that maps ΔABC onto ΔA'B'C'?

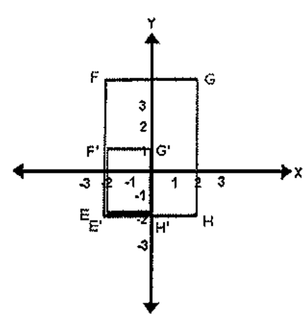
Centers of Dilation

A dilation with center, O and scale factor k, maps P onto P'.

The following properties are true:

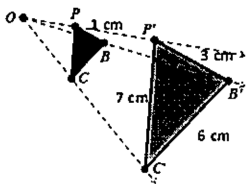




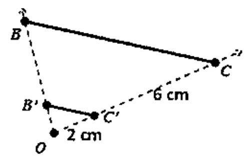


Determine whether the dilation is an enlargement or a reduction, determine the scale factor and express the ratio of the pre-image to image in simplest form (no decimals or fractions)

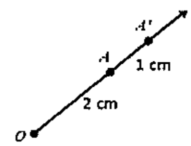
Enlarge or Reduce
______:______
Scale Factor k = ______



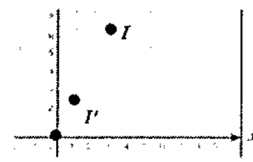
Enlarge or Reduce
______:______
Scale Factor k = ______



Enlarge or Reduce
______:______
Scale Factor k = ______

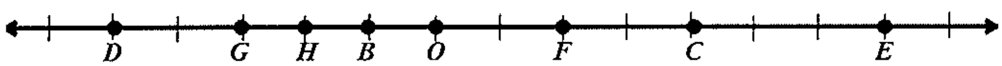


Enlarge or Reduce
______:______
Scale Factor k = ______



$D_{o,k}(P) = P'$

- o is the center
- k is the scale factor
- P is the pre-image
- P' is the image



$D_{o,2}(B)$ = ______

$D_{G,5}(H)$ = ______

$D_{o,2}(F)$ = ______

$D_{G,-2}(H)$ = ______

$D_{H,2}($ $) = F$

$D_{c,3}(F)$ = ______

$D_{B,3}(O)$ = ______

$D_{C,1/2}($ $) = F$

$D_{c,-2}(E)$ = ______

$D_{B,2}(G)$ = ______

$D_{H,1/2}(F)$ = ______

$D_{D,3/4}(E)$ = ______

G.SRT.1 WORKSHEET #2 – PATTERSON **NAME:**____________________________

1. Circle whether the following situations are REDUCTIONS OR ENLARGEMENTS.

a) Scale Factor = 7

Reduction or Enlargement

b) $D_{O,3}(H) = H'$

Reduction or Enlargement

c)

Reduction or Enlargement

d) $D_{O,1.75}(A) = A'$

Reduction or Enlargement

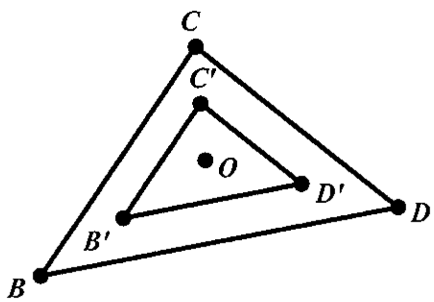
e) Scale Factor of 2/3

Reduction or Enlargement

f) $D_{O,\frac{5}{3}}(G) = G'$

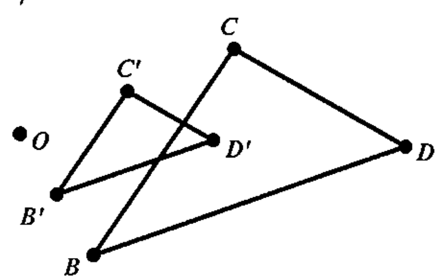
Reduction or Enlargement

g)



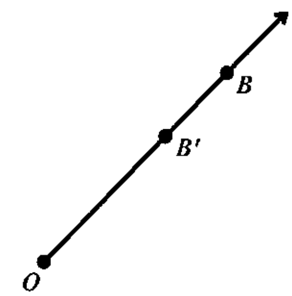
Reduction or Enlargement

h)



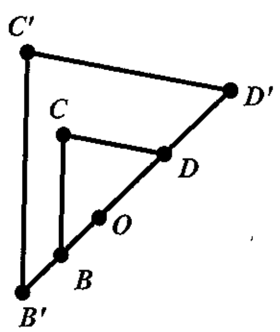
Reduction or Enlargement

i)



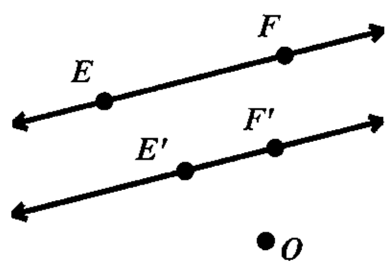
Reduction or Enlargement

J)



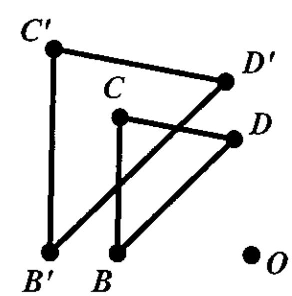
Reduction or Enlargement

k)



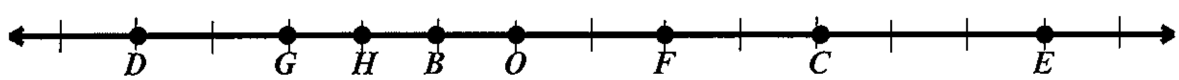
Reduction or Enlargement

l)



Reduction or Enlargement

2. Determine the missing point.



a) $D_{O,3}(B) = (\_\_\_\_\_\_)$

b) $D_{O,-2}(H) = (\_\_\_\_\_\_)$

c) $D_{G,-2}(H) = (\_\_\_\_\_\_)$

d) $D_{E,3}(C) = (\_\_\_\_\_\_)$

e) $D_{H,4}(\_\_\_\_\_\_) = (F)$

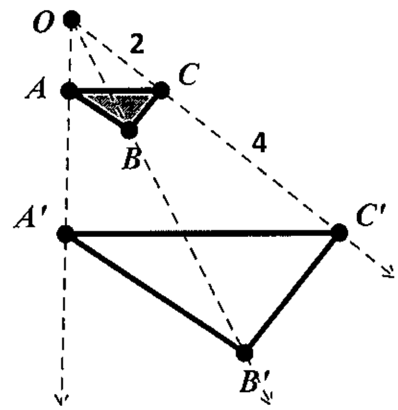
f) $D_{H,-9}(\_\_\_\_\_\_) = (E)$

g) $D_{H,3}(\_\_\_\_) = (C)$

h) $D_{C,2.5}(F) = (\_\_\_\_)$

i) $D_{G,\frac{7}{5}}(F) = (\_\_\_\_\_)$

3. Tiffany sees this given dilation and claims that the scale factor is 2 because 4 is twice as big as 2. Is this a scale factor of 2? Explain.



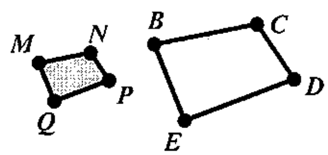
4. Determine whether the following are stretch or dilation transformations.

a)

$H(x,y) ---> (2x,5y)$

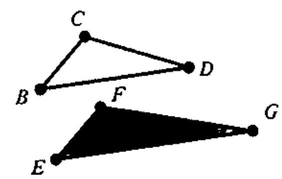
Stretch or Dilation

b)



Stretch or Dilation

c)



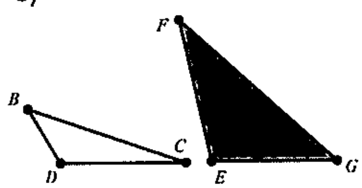
Stretch or Dilation

d)

$W(x,y) ---> (\sqrt{5}x,\sqrt{5}y)$

Stretch or Dilation

e)



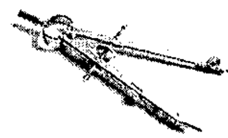
Stretch or Dilation

f)

$L(x,y) ---> (.3x,.2y)$

Stretch or Dilation

Unit 7 Lesson 2: Constructing Dilations

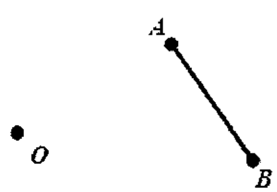


Steps:

1. Draw a straight line through the center of dilation and pre-image.
2. Measure the distance from the center of dilation to the pre-image.
3. If scale factor is an integer, copy the segment as many times as you need.
   (positive moves on same side as pre-image, negative moves on opposite side of center)
4. If scale factor includes 1/2, 1/4, 3/4 use the perpendicular bisector construction.
5. Repeat as many times as necessary!

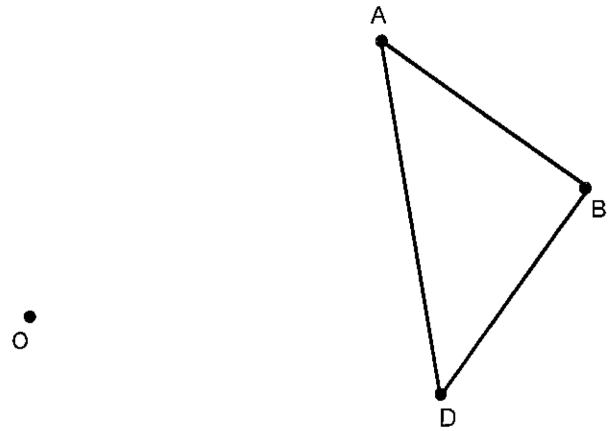
Example 1:

$D_{O,3}(\overline{AB})$



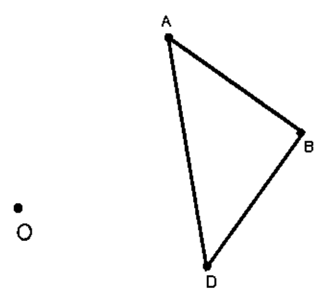
Example 2:

$D_{O,\frac{1}{2}}(\Delta ADB)$



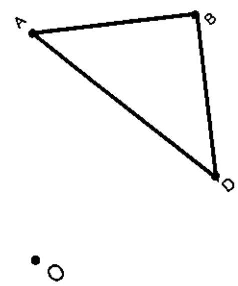
Example 3:

$D_{O,-1}(\Delta ADB)$



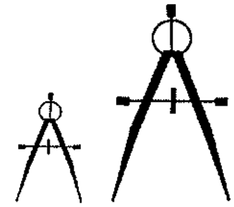
Example 4:

$D_{o, -2.25}(\Delta \text{ADB})$

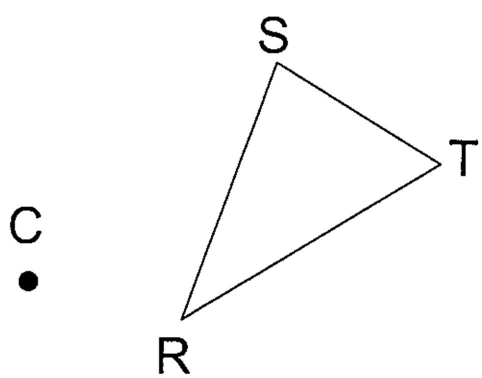


Constructing a Dilation

Name______________________________



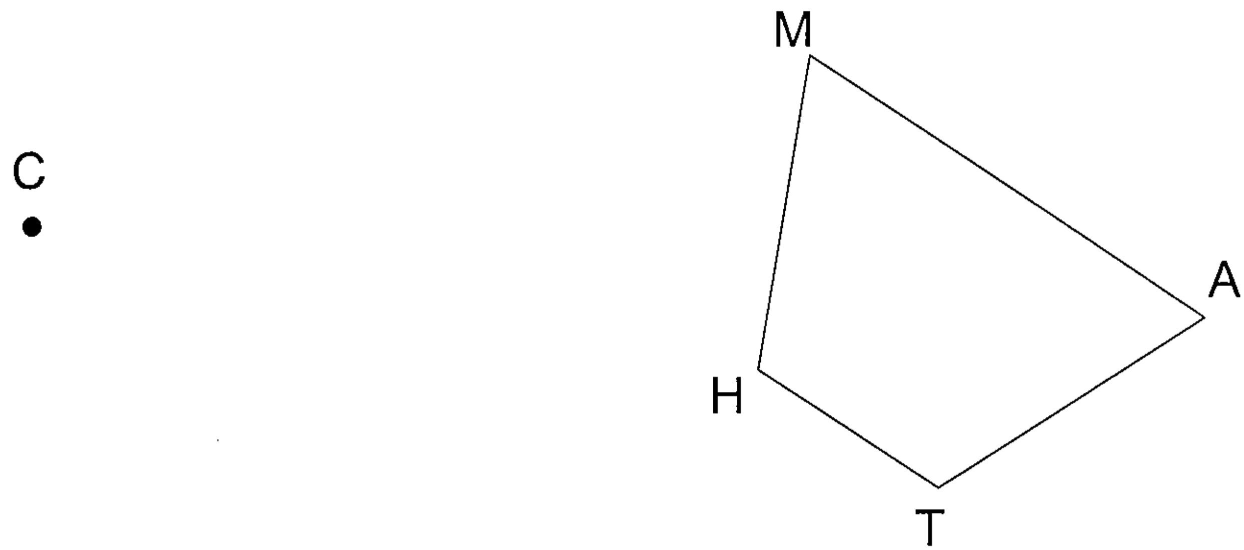
1. Construct the image of ΔRST after a dilation centered at C with a scale factor of 2.



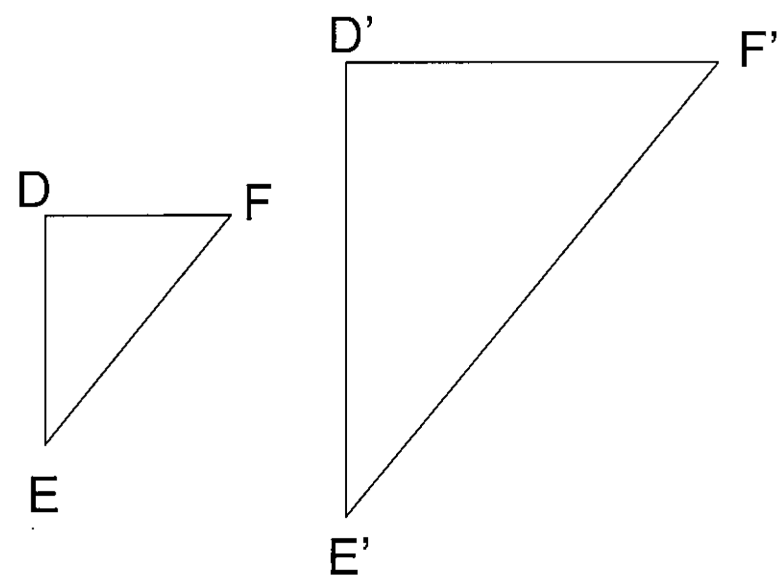
Steps:

1. Draw ray $\overline{CR}$.
2. Measure $\overline{CR}$. Copy $\overline{CR}$ on the ray to the right of R. Label this point R'.
3. Repeat this process for the vertices S and T. Label S' and T'.
4. Draw $\Delta R'S'T'$, which is the dilation of ΔRST, centered in C, with a scale factor of 2.

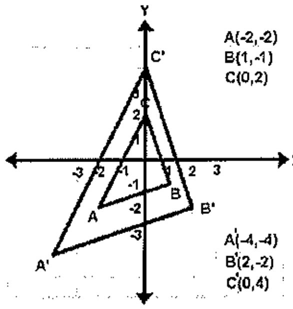
2. Construct a dilation of *MATH*, centered at *C*, with a scale factor of ½ .

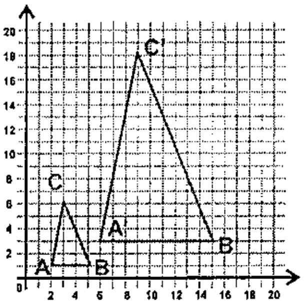


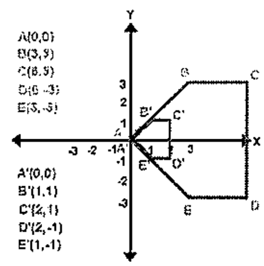
3. Using constructions, find the center of the dilation shown below which has a scale factor of 2. Label the center *C*.



Unit 7 Lesson 3
Dilations with Coordinates (Center at Origin)

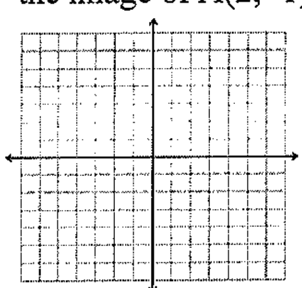




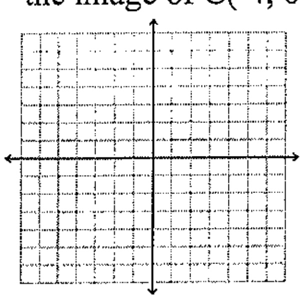


$$D_{O,\,k}\,(x, y) = (kx, ky)$$

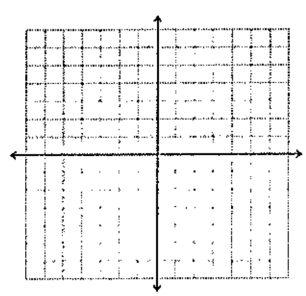
Example 1:
What are the coordinates of A',
the image of A(2, -1) after $D_{O,\,3}$?



Example 2:
What are the coordinates of C',
the image of C(-4, 6) after $D_{O,\,1/2}$?

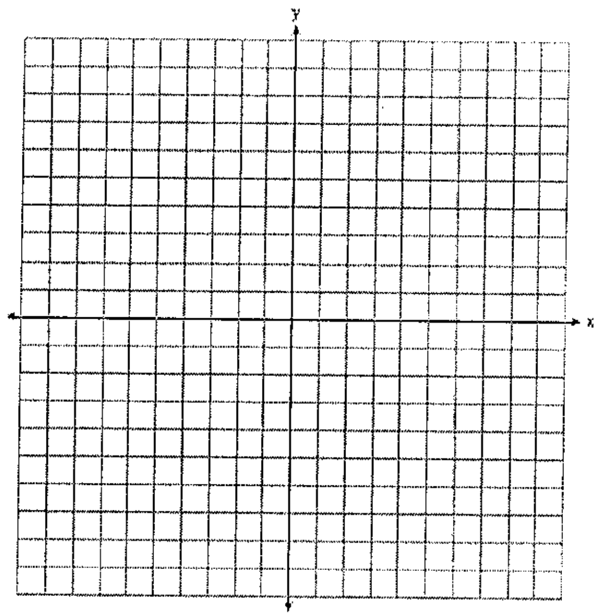


Example 3:
What are the coordinates of B',
the image of B(3, -2) after $D_{O,\,-2}$?



Example 4:

Triangle ABC has vertices $A(5,1)$, $B(1,4)$ and $C(1,1)$. State and label the coordinates of the vertices of $\Delta A''B''C''$, the image of ΔABC, following the composite transformation $T_{1,-1} \circ D_{O,2}$.



Example 5:

The coordinates of A' after a dilation of 3 with respect to the origin are (12, -3). What are the coordinates of A?

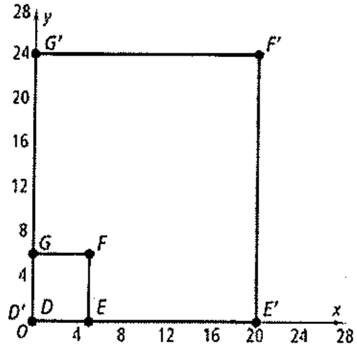
Example 6:

The coordinates of B' after a dilation of 1/2 with respect to the origin are (3, -8) What are the coordinates of B?

Example 7:

The point P(-6, 8) maps onto P'(3, -4) under a certain dilation with center at the origin. What are the coordinates of (-2, 6) under the same dilation?

Example 8 $D_{O,?}(DEFG) = D'E'F'G'$



a) What is the constant of dilation (scale factor)?

b) What is the perimeter of ABCD?

c) What is the perimeter of A'B'C'D'?

d) What is the area of ABCD?

e) What is the area of A'B'C'D'?

f) Do the angle measures change?

Example 9

A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

1 The area of the image is nine times the area of the original triangle.
2 The perimeter of the image is nine times the perimeter of the original triangle.
3 The slope of any side of the image is three times the slope of the corresponding side of the original triangle.
4 The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

Example 10

If $\triangle ABC$ is dilated by a scale factor of 3, which statement is true of the image $\triangle A'B'C'$?

1 $3A'B' = AB$
2 $B'C' = 3BC$
3 $m\angle A' = 3(m\angle A)$
4 $3(m\angle C') = m\angle C$

Date ___________

Dilations with Coordinates (**Centered at the Origin**) HOMEWORK

Geometry CC

1 What are the coordinates of the point (2,−4) under the dilation $D_{O,2}$?

1) (8,−4)
2) (4,−8)
3) (−8,4)
4) (−4,8)

2 What are the coordinates of point (−1,4) under dilation $D_{O,-2}$?

1) (−2,8)
2) (2,−8)
3) (−8,2)
4) (8,−2)

3 Find the image of (3,−2) under the dilation $D_{O,3}$.

4 What is the image of point *A*(1,3) after a dilation with the center at the origin and a scale factor of 4?

5 If *P*(4,−3) is transformed under the dilation $D_{O,-3}$, what is the image of *P*′?

6 Find the image of *A*(−3,2) under a dilation with the center at the origin and a scale factor of -1.

7 Triangle *ABC* has vertices *A*(6,6), *B*(9,0), and *C*(3,−3). State and label the coordinates of $\Delta A'B'C'$, the image of ΔABC after a $D_{O,1/3}$.

8 In which quadrant would the image of point (5,−3) fall after a dilation, with center at the origin using a factor of -4?

9 In which quadrant will the image of *A*(4,−2) lie after dilation $D_{O,-2}$?

10 The image of point *A* after a dilation of 3 with center at the origin (6,15). What was the original location of point *A*?

1) (2,5)
2) (3,12)
3) (9,18)
4) (18,45)

12 The point *A*(6,3) maps onto *A*′(2,1) under a dilation with respect to the origin. What is the constant of dilation?

1) $\frac{1}{3}$
2) $\frac{1}{2}$
3) 3
4) −2

13 Under a dilation with respect to the origin, the image of $P(-15,6)$ is $P'(-5,2)$. What is the constant of dilation?

1) -4
2) $\frac{1}{3}$
3) 3
4) 10

14 If the dilation $D_{O,k}(-2, 4)$ equals $(1,-2)$, the scale factor k is equal to

1) $\frac{1}{2}$
2) 2
3) $-\frac{1}{2}$
4) -2

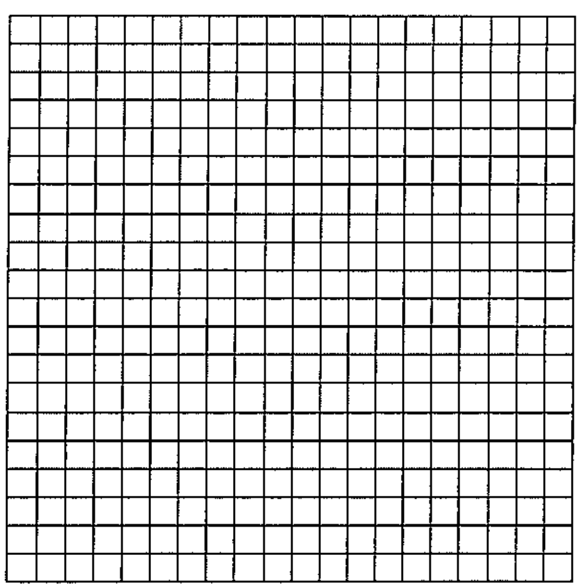
15 Under a dilation where the center of dilation is the origin, the image of $A(-2,-3)$ is $A'(-6,-9)$. What are the coordinates of B', the image of $B(4,0)$ under the same dilation?

1) $(-12,0)$
2) $(12,0)$
3) $(-4,0)$
4) $(4,0)$

16 Triangle $A'B'C'$ is the image of ΔABC after a dilation of 2. Which statement is true?

1) $AB = A'B'$
2) $BC = 2(B'C')$
3) $m\angle B = m\angle B'$
4) $m\angle A = \frac{1}{2}(m\angle A')$

17 Farmington, New York, has plans for a new triangular park. If plotted on a coordinate grid, the vertices would be $A(3,3)$, $B(5,-2)$, and $C(-3,-1)$. However, a tract of land has become available that would enable the planners to increase the size of the park, which is based on the following transformation of the original triangular park, $R_{O,270} \circ D_{O,2}$. On the grid below, graph and label both the original park ΔABC and its image, the new park $\Delta A''B''C''$, following the transformation.



Unit 7 Lesson 4
Dilations when Center is NOT THE ORIGIN

- A dilation moves a point either away or towards the center of dilation.
- Center of dilation can be inside figure, outside figure, on the figure.
- Scale factors:

k >1 enlargement (example k = 2)

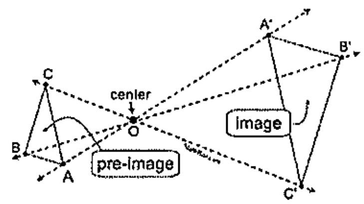
0 < k < 1 reduction (example k = ½)

* Center of dilation will be **at the end of** pre-image and image points

k < -1 enlargement and rotation 180° (example k = -2)

-1 < k < 0 reduction and rotation 180° (example k = - ½)

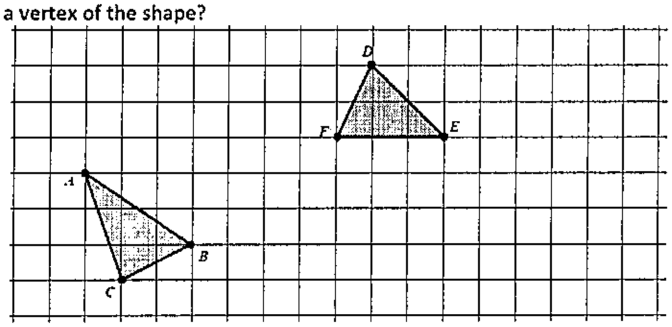
* Center of dilation will be **between** pre-image and image points



1. What happens when the center of dilation is a vertex of the shape?

a) Dilate ΔABC from C using a scale factor of 2
$D_{C,2}(\Delta ABC)$

b) Dilate ΔDEF from D using a scale factor of 3
$D_{D,3}(\Delta DEF)$



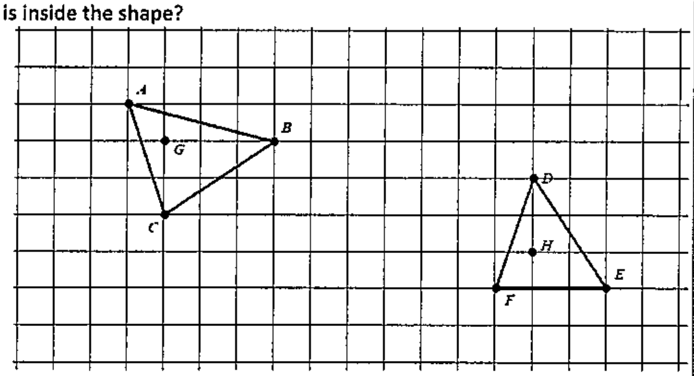
2. What happens when the center of dilation is inside the shape?

a) Dilate ΔABC from G using a scale factor of 3

$D_{G,3}(\Delta ABC)$

b) Dilate ΔDEF from H using a scale factor of 2

$D_{H,2}(\Delta DEF)$



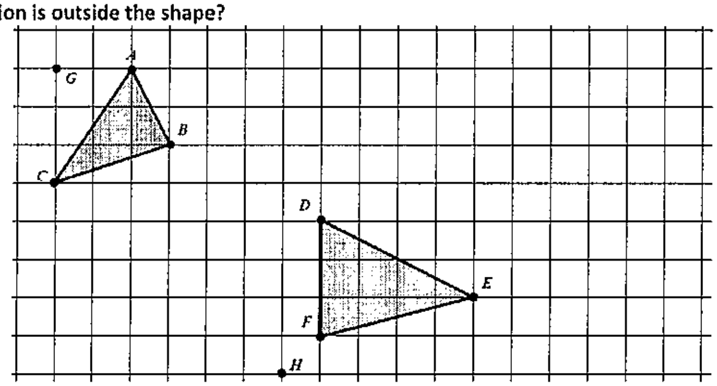
3. What happens when the center of dilation is outside the shape?

a) Dilate ΔABC from G using a scale factor of 2

$D_{G,2}(\Delta ABC)$

b) Dilate ΔDEF from H using a scale factor of 2

$D_{H,2}(\Delta DEF)$



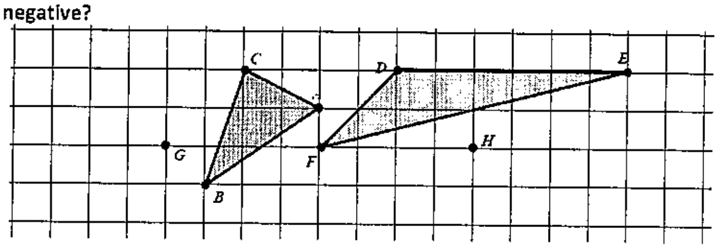
4. What happens when the scale factor is negative?

a) Dilate ΔABC from G using a scale factor of -1

$D_{G,-1}(\Delta ABC)$

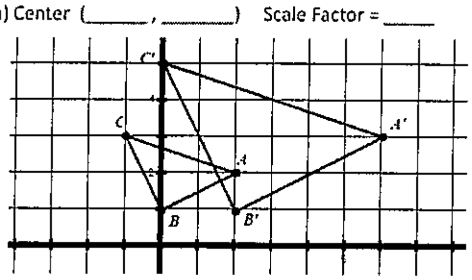
b) Dilate ΔDEF from H using a scale factor of -½

$D_{H,-\frac{1}{2}}(\Delta DEF)$

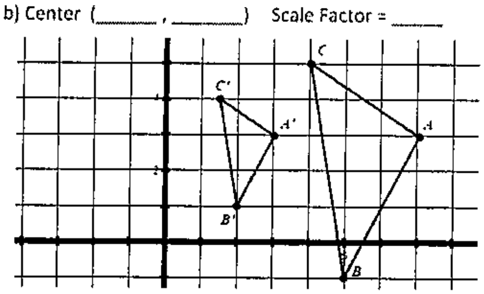


5. Work backwards to find the center of dilation, and also determine the scale factor.

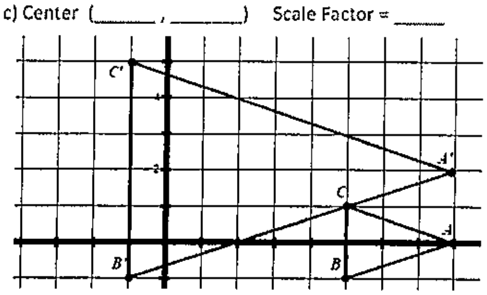
a) Center (______, ______) Scale Factor = _____



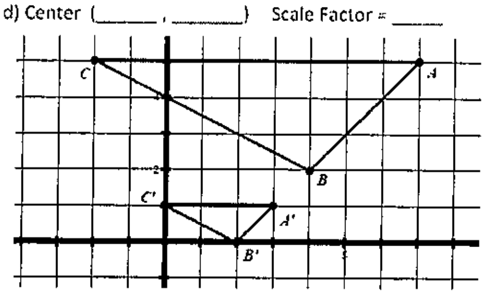
b) Center (______, ______) Scale Factor = _____



c) Center (______, ______) Scale Factor = _____



d) Center (______, ______) Scale Factor = _____



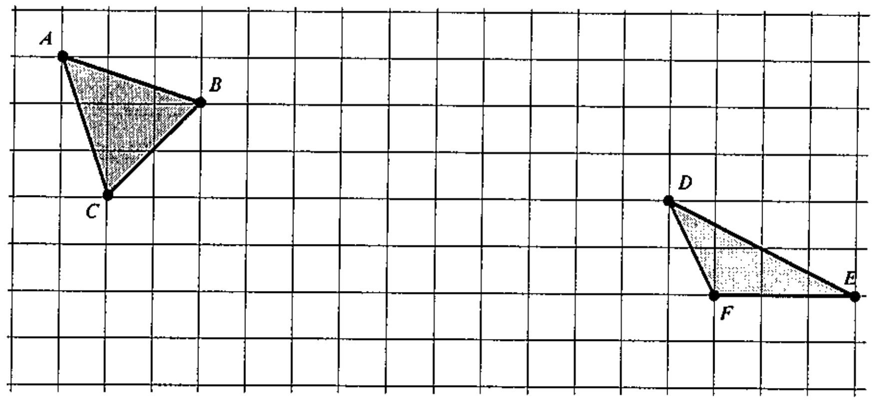
1. What happens when the center of dilation is a vertex of the shape?

a) Dilate ΔABC from A using a scale factor of 2

$D_{A,2}(\Delta ABC)$

b) Dilate ΔDEF from E using a scale factor of 3

$D_{E,3}(\Delta DEF)$



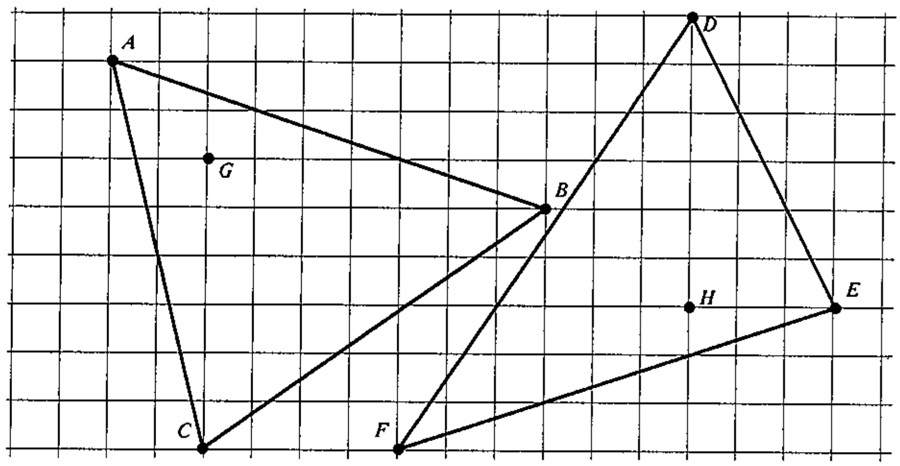
2. What happens when the center of dilation is inside the shape?

a) Dilate ΔABC from G using a scale factor of ½

$D_{G,\frac{1}{2}}(\Delta ABC)$

b) Dilate ΔDEF from H using a scale factor of $\frac{1}{3}$

$D_{H,\frac{1}{3}}(\Delta DEF)$



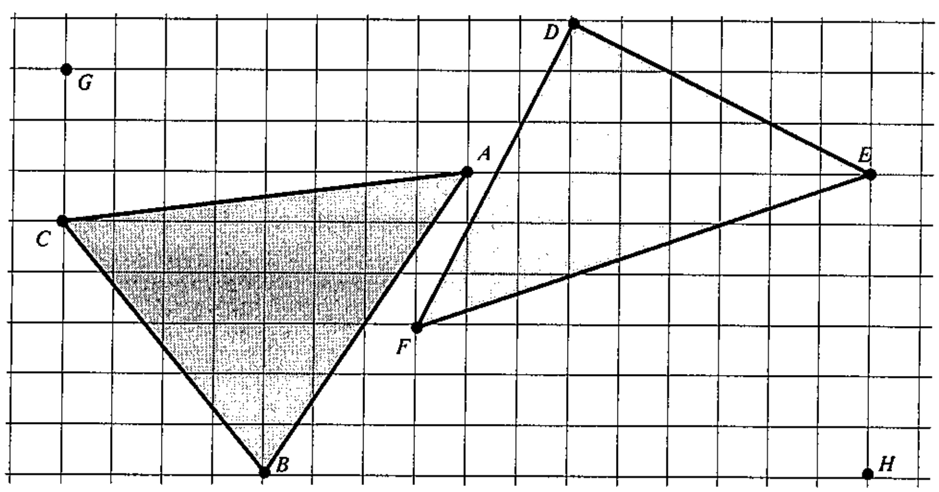
3. What happens when the center of dilation is outside the shape?

a) Dilate ΔABC from G using a scale factor of ½

$D_{G,\frac{1}{2}}(\Delta ABC)$

b) Dilate ΔDEF from H using a scale factor of $\frac{1}{3}$

$D_{H,\frac{1}{3}}(\Delta DEF)$



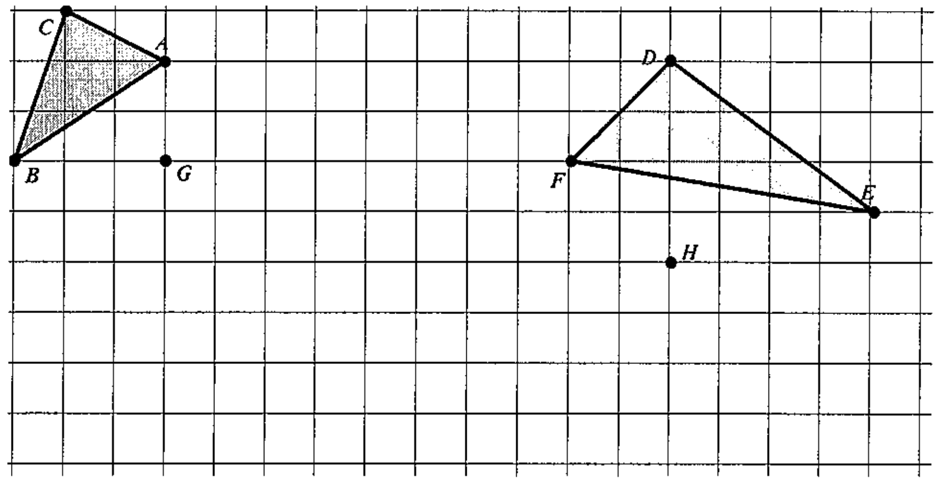
4. What happens when the scale factor is negative?

a) Dilate ΔABC from G using a scale factor of-2

$D_{G,-2}(\Delta ABC)$

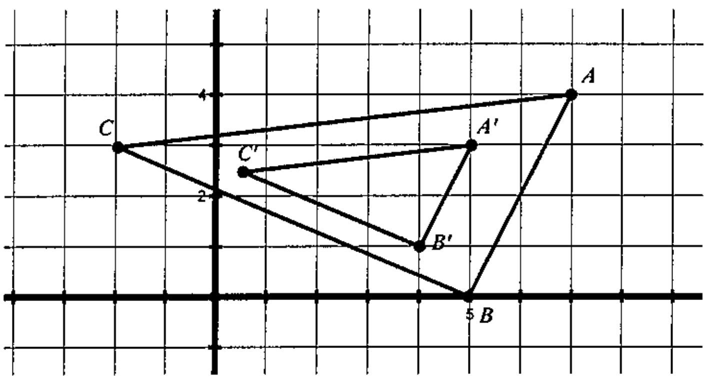
b) Dilate ΔDEF from H using a scale factor of -1

$D_{H,-1}(\Delta DEF)$

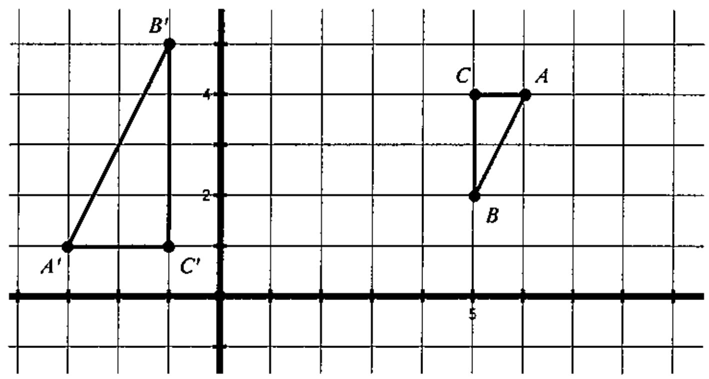


5. Work backwards to find the center of dilation, and also determine the scale factor.

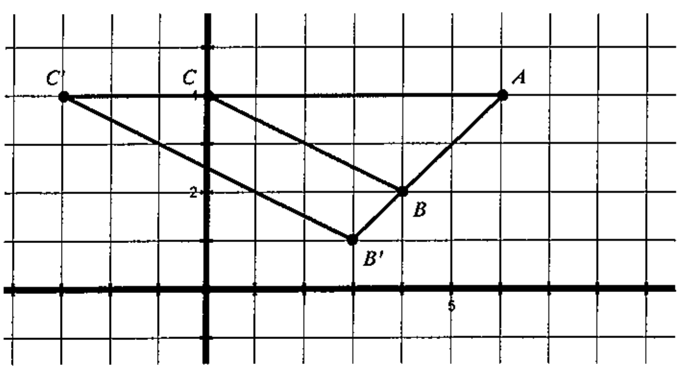
a) Center (______, _______) Scale Factor = _____



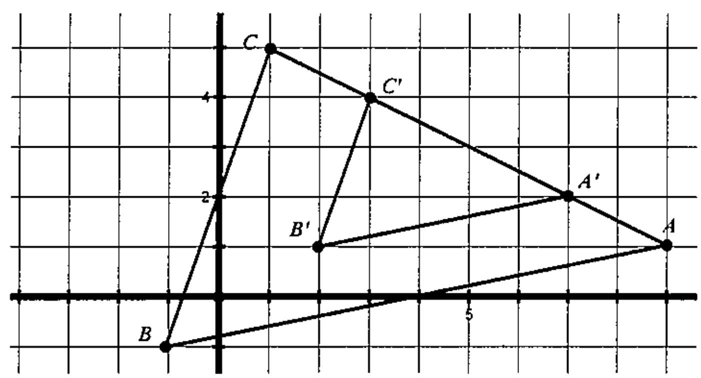
b) Center (______, _______) Scale Factor = _____



c) Center (______, _______) Scale Factor = _____



d) Center (______, _______) Scale Factor = _____



Unit 7 Lesson 5: Dilating Lines

Review:

3 Forms of an equation of a line:

$y = mx + b$

$ax + by = c$

$y - y_1 = m(x - x_1)$

Examples:

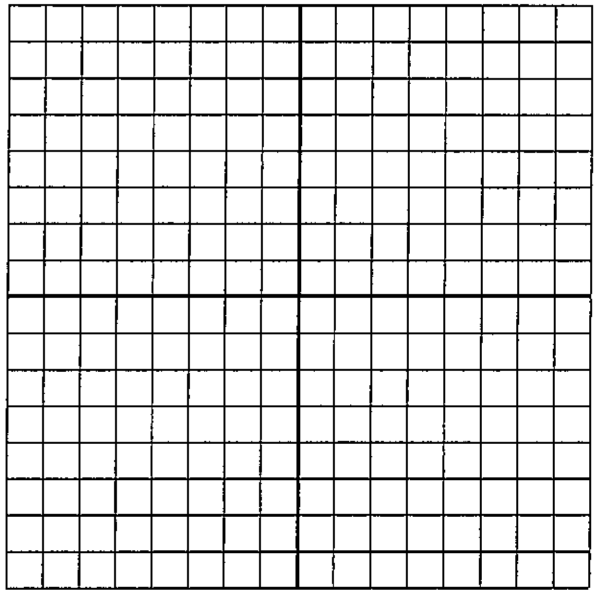
Graph each line

1. $y = 2/3\ x + 3$

2. $2y = -x + 4$

3. $2x + y = -6$

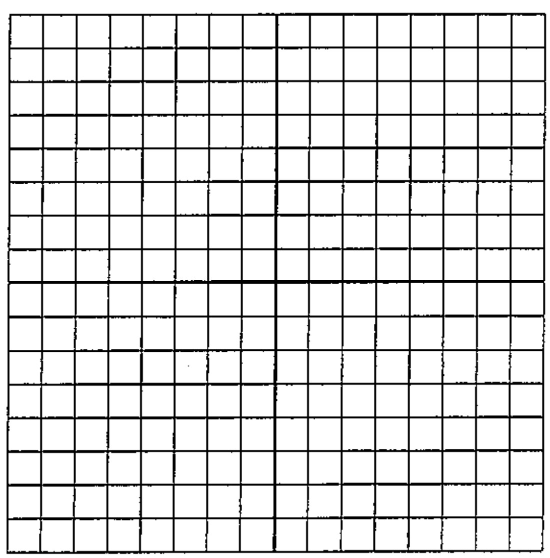
4. $3x - 6y = 12$



Dilating Lines

Example 5

The equation of line h is $2x + y = 1$. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m?



Example 6

The line $y = 2x - 4$ is dilated by a scale factor of 3/2 and centered at the origin.
What is an equation if the image after the dilation?

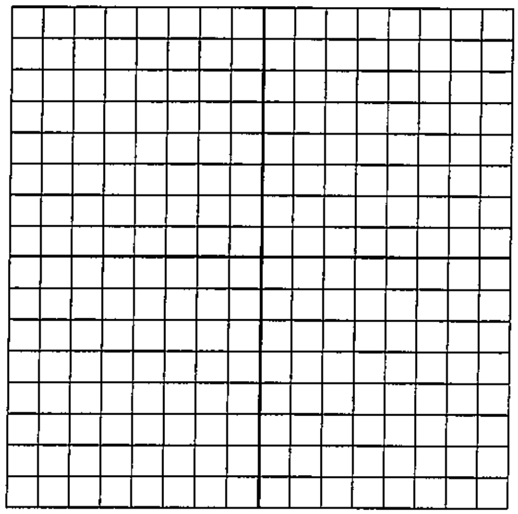
Example 7

The line $3y = -2x + 8$ is transformed by a dilation centered at the origin. Which linear equation could be its image?

1 $2x + 3y = 5$
2 $2x - 3y = 5$
3 $3x + 2y = 5$
4 $3x - 2y = 5$

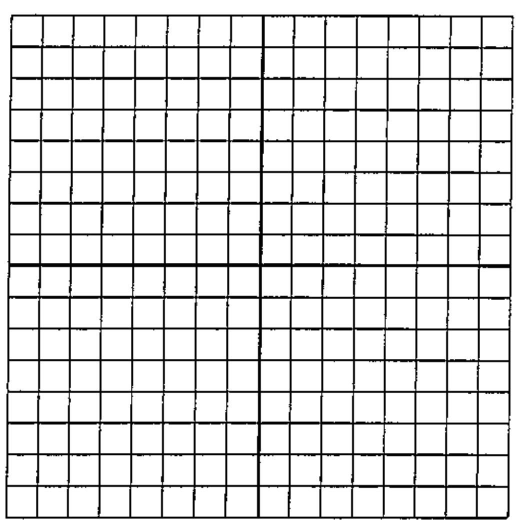
Example 8 Center Not Origin

Write the equation of the line $y = 3x + 2$ after a dilation of 2 with respect to (3, 1).



Example 9 Center Not Origin

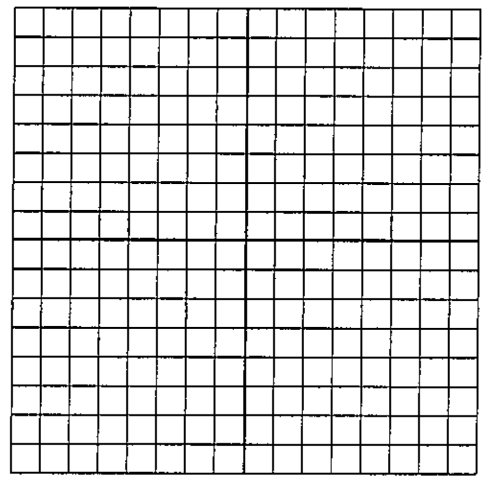
Write the equation of the line $2x + y = 8$ after a dilation of 1/2 with respect to (-1, 4).



Example 10 Center Not Origin

Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at (3,8). The line's image is

1 $y = 3x - 8$
2 $y = 3x - 4$
3 $y = 3x - 2$
4 $y = 3x - 1$



Date ___________

Geometry CC

Dilating Lines

Express all answers in $y = mx + b$ format.

1. Write the equation of the line $y = \frac{1}{2}x + 4$ after a dilation of 2 with respect to the origin.

2. Write the equation of the line $y = 2/3\ x - 9$ after a dilation of 1/3 with respect to the origin.

3. Write the equation of the line $2x + 3y = 18$ after a dilation of 2/3 with respect to the origin.

4. Write the equation of the line $y = \frac{1}{2}x + 4$ after a dilation of ½ with center (2, 1). USE GRAPH PAPER

5. Write the equation of the line $y = 2/3\ x - 9$ after a dilation of 1/3 with respect to the point (6, - 4). USE GRAPH PAPER

6. Write the equation of the line $2x + 3y = 18$ after a dilation of 2 with center (1, 2). USE GRAPH PAPER

Date ________ Geometry CC

Unit 2-7 Review: Dilations

1) Which of the following is a stretch?

A) $T(x,y) ----> (-x,-y)$ B) $T(x,y) ----> (x+7,y-5)$

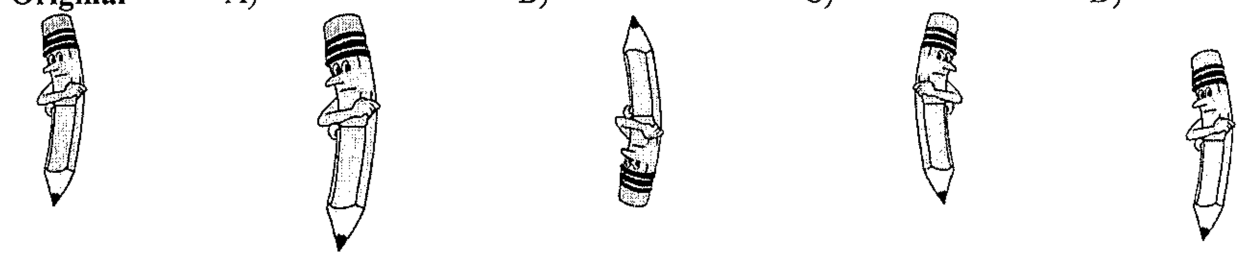
C) $T(x,y) ----> (2x,2y)$ D) $T(x,y) ----> (1x,4y)$

2) Which of the following is not a rigid motion?

A) Stretch B) Translation C) Rotation D) Reflection

3) Given the original figure, which of the following is a dilation?

Original A) B) C) D)

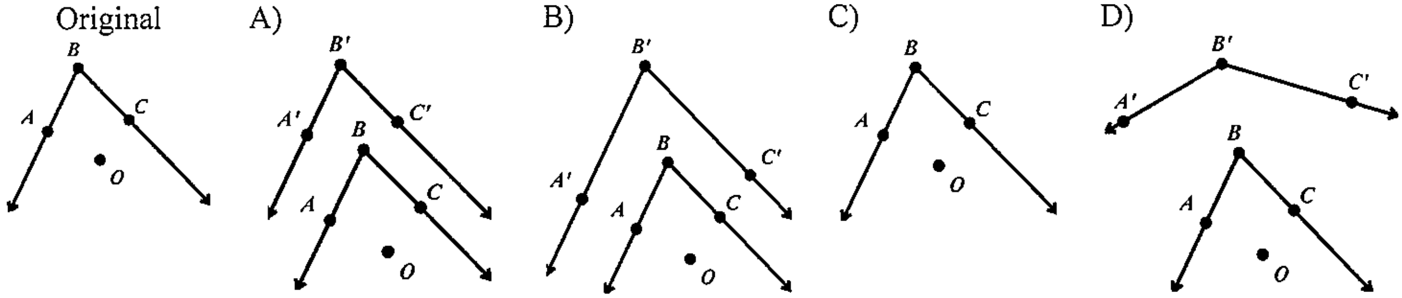


4) Which of the following ratios of pre-image : image represents an enlargement?

A) 1 : 1.00002 B) 5 : 4 C) 0.5 : 0.088 D) 7 : 6.5

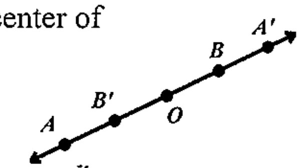
5) If we $D_{O,2}$ then the correct diagram would be:

Original A) B) C) D)

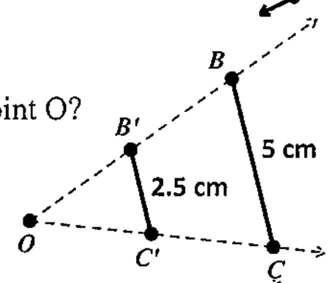


6) Determine the scale factor that best suits the provided diagram (O is the center of dilation).

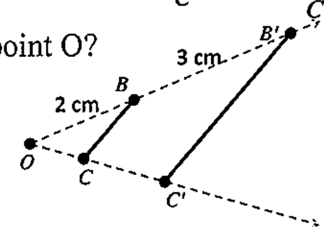
A) 2 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) -1



7) Determine the scale factor of the given dilation from point O?



8) Determine the scale factor of the given dilation from point O?



9) Given $D_{O,-4}P(x,y)=P'(4,8)$ then P(x,y) is

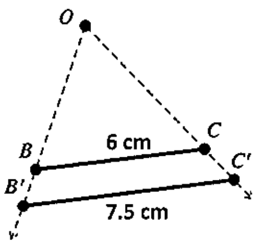
A) P(-1,-2) B) P(4,-32) C) P(4,4) D) P(-16,-32)

10) Determine whether the dilation is an enlargement or a reduction.
Determine the ratio of pre-image to image in the most reduced form (no decimals).
Determine the scale factor, k.

a) Enlarge or Reduce

_____ : _____

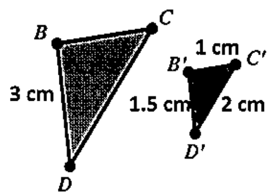
k = _______



b) Enlarge or Reduce

_____ : _____

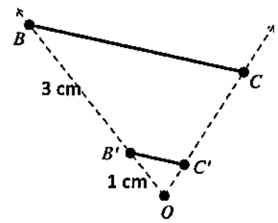
k = _______



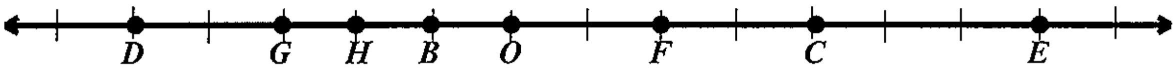
c) Enlarge or Reduce

_____ : _____

k = _______



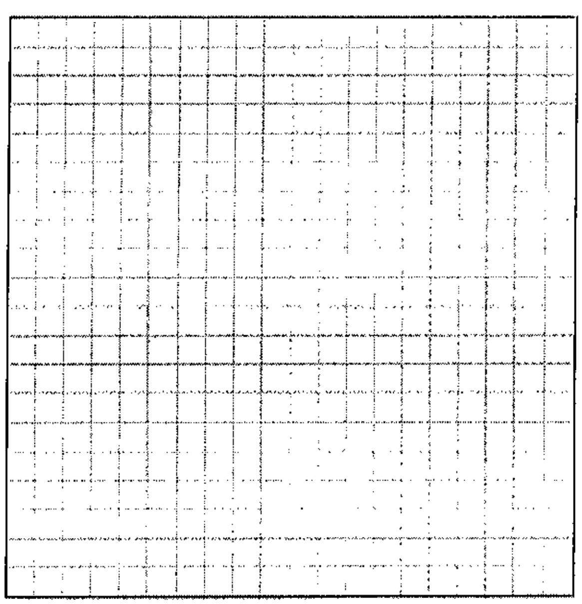
11) Determine the point.



a) $D_{H,4}(B)=(\_\_\_\_\_\_)$ b) $D_{C,\frac{1}{2}}(\_\_\_\_\_\_)=(F)$ c) $D_{H,-2}(G)=(\_\_\_\_\_\_)$

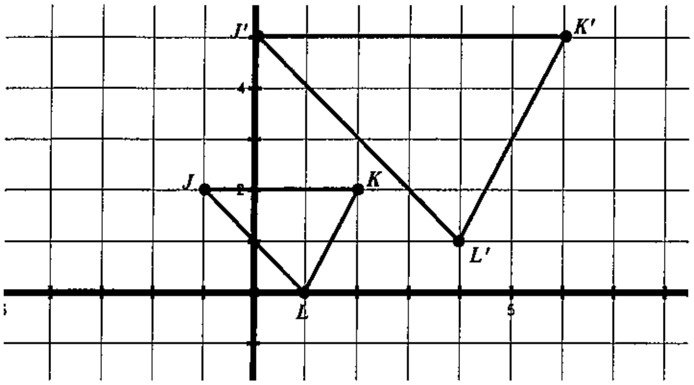
d) $D_{H,-\frac{1}{3}}(E)=(\_\_\_\_\_\_)$ e) $D_{D,\frac{3}{2}}(G)=(\_\_\_\_)$

12) The coordinates of the vertices of ΔABC $A(1,3)$, $B(-2,2)$ and $C(0,-2)$. On the grid below, graph and label $\Delta A''B''C''$, the result of the composite transformation $D_2 \circ T_{3,-2}$. State the coordinates of A'', B'', and C''. The center of the dilation is the origin

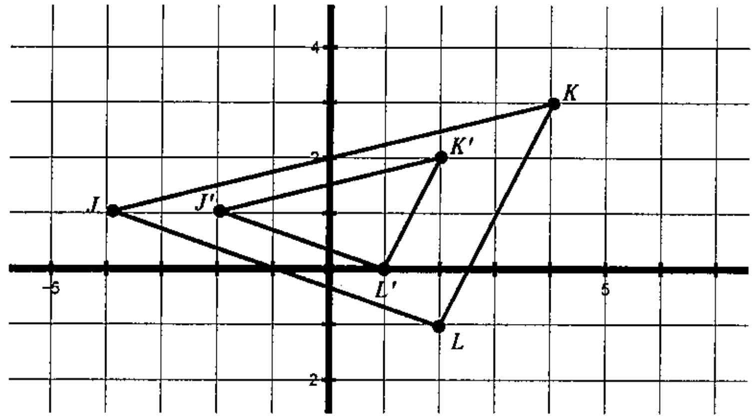


13) Work backwards to find the center of dilation, and also determine the scale factor.

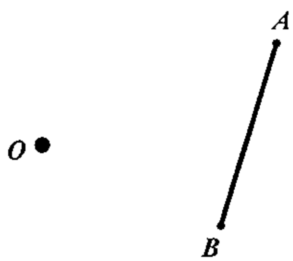
Center (______ , ________) Scale Factor = _____



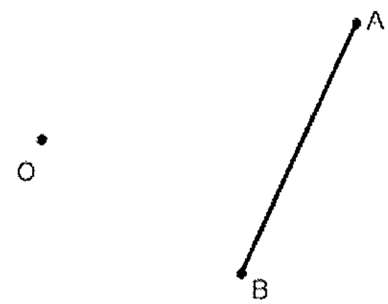
Center (______ , ________) Scale Factor = _____



14) Use a compass and a straightedge to construct $D_{O,2}\left(\overline{AB}\right)$



15) Use a compass and a straightedge to construct $D_{O,\frac{1}{2}}(AB)$



16. What would be the equation of the line $2x + y = 6$ after a dilation of 3 centered about the origin?

17. What is the equation of $y = \frac{2}{3}x - 2$ after D_3 with respect to the point (-3, 1)? USE GRAPH PAPER

18. What is the equation of $y = 3x - 4$ after $D_{1/2}$ with respect to the point (2, 2)? USE GRAPH PAPER

19. A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?

1 9 inches
2 2 inches
3 15 inches
4 18 inches