

Geometry

Unit 2-5

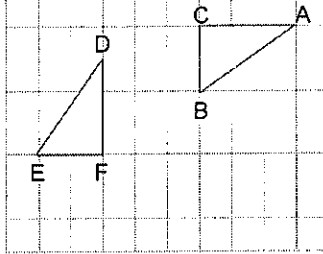
Congruent Triangles

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Unit 5 Lesson 1 CONGRUENCE

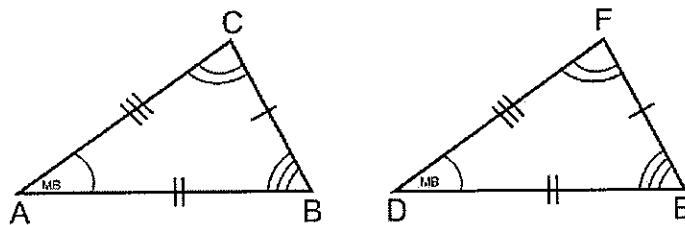
Congruent Symbol: \sim means same shape (similarity)
 \cong means same size (equality)

Definition: The word "congruent" is defined in the dictionary as "identical in form; coinciding exactly when superimposed."
 The origin of the word "congruent" is from the Latin to "agree" or "to meet together".



It is possible to turn, flip and/or slide one figure so it will fit exactly on the other figure.
 The areas of congruent figures are equal.
 The perimeters of congruent figures are equal.
 The corresponding angles of congruent figures are equal in measure.
 The corresponding line segments of congruent figures are equal in length.

Corresponding Parts



$$\triangle ABC \cong \triangle DEF$$

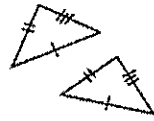
Two triangles are congruent if they have:

- 3 pairs of \cong sides
- 3 pairs of \cong angles

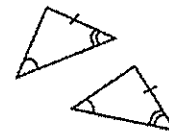
But we don't have to know all three sides and all three angles ... usually three out of the six is enough.

Methods that DO prove triangles congruent.

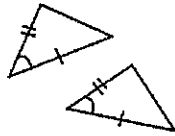
SSS
Side-Side-Side



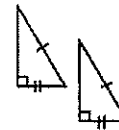
AAS (or SAA)
Angle-Angle-Side



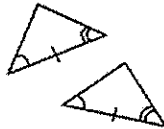
SAS
Side-Angle-Side



HL
Hypotenuse-Leg



ASA
Angle-Side-Angle



Methods that DO NOT prove triangles congruent.

AAA
Angle-Angle-Angle

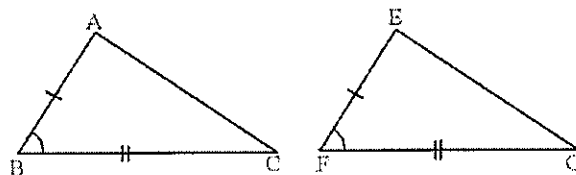
SSA or ASS
Side-Side-Angle

<https://www.youtube.com/watch?v=vGuiy7NnJIM>

Once triangles are proven congruent, the corresponding leftover "parts" that were not used in SSS, SAS, ASA, AAS and HL, are also congruent.

CPCTC

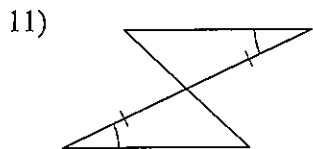
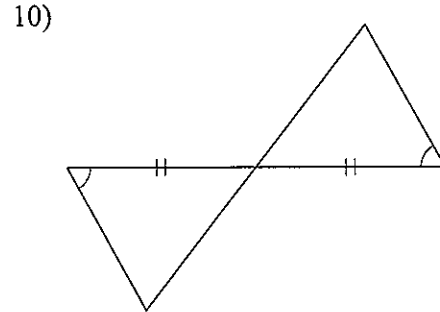
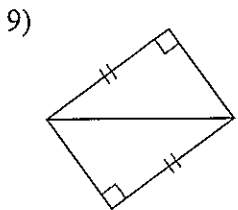
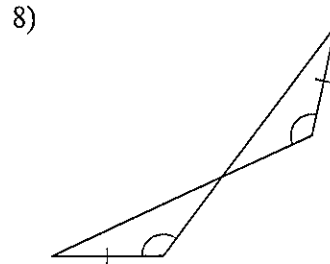
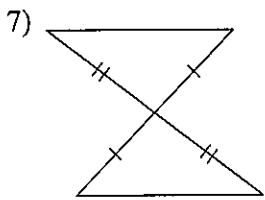
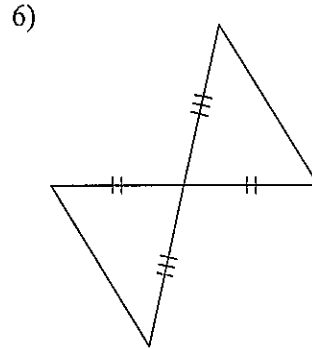
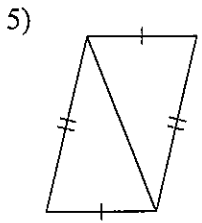
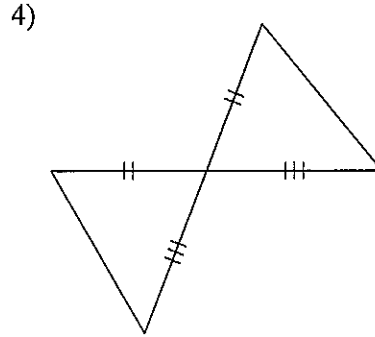
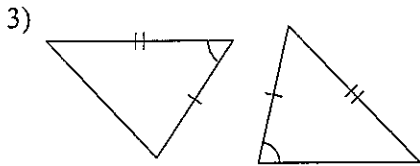
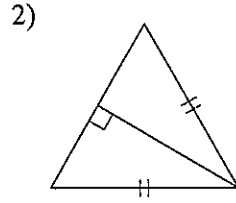
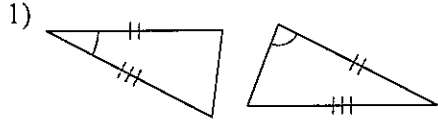
Corresponding Parts of Congruent Triangles are Congruent.



SSS, SAS, ASA, AAS, and HL Congruence

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State if the two triangles are congruent. If they are, state how you know.

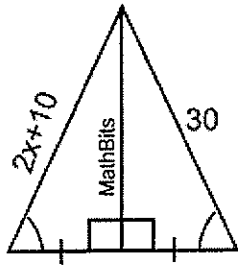


Congruent Triangles

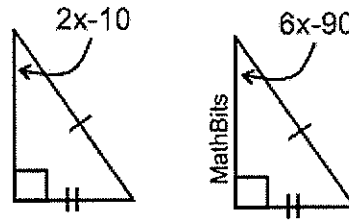
Name _____

Directions: First, indicate the method (SSS, SAS, ASA, AAS, HL) used to prove these triangles congruent based upon the ways the diagrams are marked. Then solve for the indicated variable.

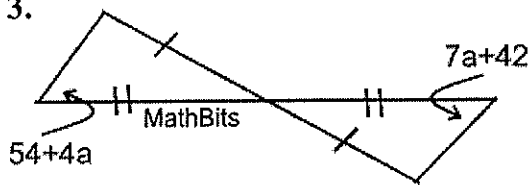
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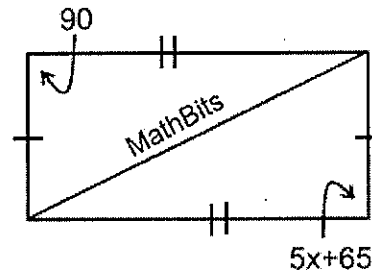
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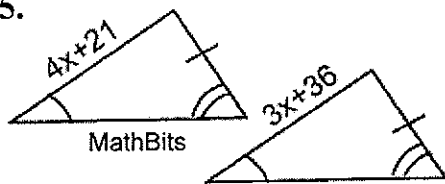
3.



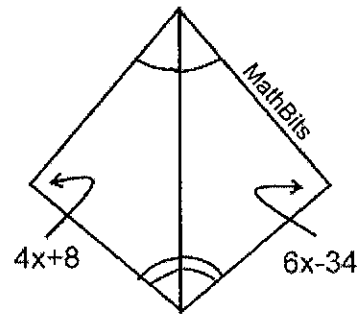
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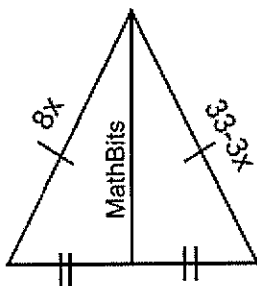
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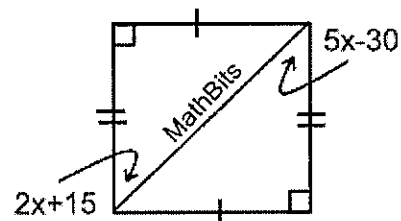
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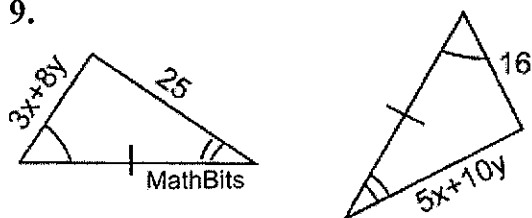
7.



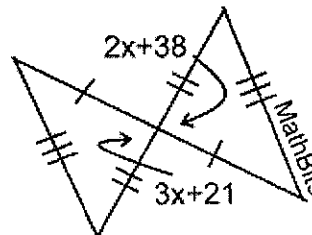
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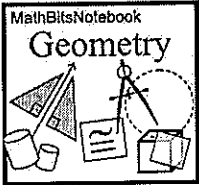


9.



10.





Recognizing Congruent Triangles

Name _____

Directions: Mark the diagrams and determine which method will prove the triangles congruent.

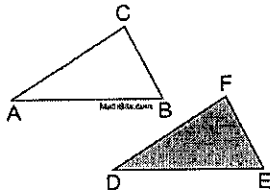
1.

Given: $\overline{AC} \cong \overline{DF}$

$\angle A \cong \angle D$

$\angle B \cong \angle E$

Prove: $\triangle ABC \cong \triangle DEF$



Choose:

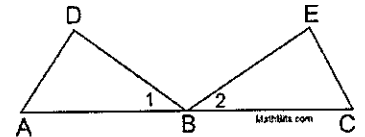
- SSS
 SAS
 ASA
 AAS
 HL
 None

2. Given: $\overline{AB} \cong \overline{CE}$; $\angle A \cong \angle C$

$\angle 1 \cong \angle 2$

B midpoint \overline{AC}

Prove: $\triangle ABD \cong \triangle CBE$



Choose:

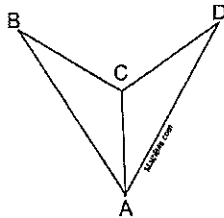
- SSS
 SAS
 ASA
 AAS
 HL
 None

3.

Given: $\overline{AB} \cong \overline{AD}$

$\overline{BC} \cong \overline{DC}$

Prove: $\triangle ABC \cong \triangle ADC$



Choose:

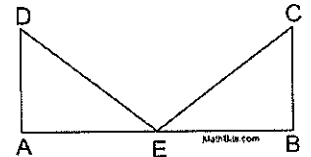
- SSS
 SAS
 ASA
 AAS
 HL
 None

4.

Given: $\overline{AD} \cong \overline{BC}$

$\overline{DE} \cong \overline{CE}$

Prove: $\triangle ADE \cong \triangle BCE$

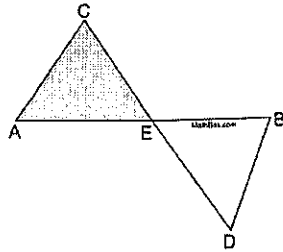


Choose:

- SSS
 SAS
 ASA
 AAS
 HL
 None

5.

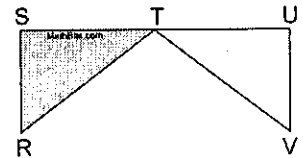
Given: $\overline{AEB}, \overline{CED}$
 \overline{CD} bisects \overline{AB}
 \overline{AB} bisects \overline{CD}
 Prove: $\triangle ACE \cong \triangle BDE$



Choose:
 SSS SAS ASA AAS HL None

6.

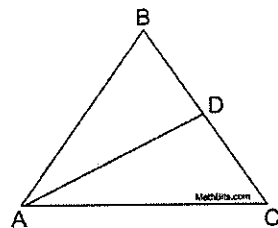
Given: $\overline{RS} \perp \overline{ST}$
 $\overline{VU} \perp \overline{TU}$
 $\overline{RT} \cong \overline{VT}$
 Prove: $\triangle RST \cong \triangle VUT$



Choose:
 SSS SAS ASA AAS HL None

7.

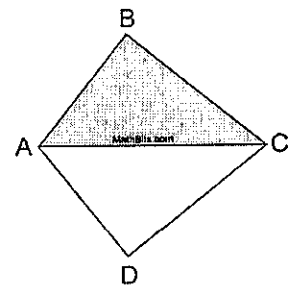
Given: \overline{AD} bisects $\angle BAC$
 $\overline{AD} \perp \overline{BC}$
 Prove: $\triangle ABC \cong \triangle ADC$



Choose:
 SSS SAS ASA AAS HL None

8.

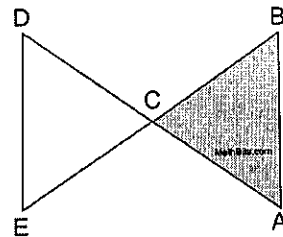
Given: $\overline{AB} \perp \overline{BC}$
 $\overline{AD} \perp \overline{DC}$
 $\overline{AB} \cong \overline{AD}$
 Prove: $\triangle ABC \cong \triangle ADC$



Choose:
 SSS SAS ASA AAS HL None

9.

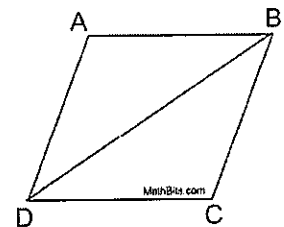
Given: $\overline{DCA}, \overline{ECB}$
 $\overline{DC} \cong \overline{AC}$
 $\overline{DE} \cong \overline{AB}$
 Prove: $\triangle DEC \cong \triangle ABC$



Choose:
 SSS SAS ASA AAS HL None

10.

Given: $\overline{AB} \parallel \overline{DC}$
 $\overline{AB} \cong \overline{CD}$
 Prove: $\triangle ABD \cong \triangle CDB$



Choose:
 SSS SAS ASA AAS HL None

Proof Reasons

SEGMENTS

1. A **segment bisector** divides a segment into 2 congruent parts.
2. A **midpoint** divides a segment into 2 congruent parts.
3. A **median** is a segment drawn from a vertex to the midpoint of the opposite side. (Then use #1)
4. Reflexive property (a segment is congruent to itself)

ANGLES

5. An **angle bisector** divides an angle into 2 congruent parts.
6. **Perpendicular lines** form right angles. (then use #7) OR (if using HL, go directly to # 11)
7. All **right angles** are congruent.
8. An **altitude** is a segment drawn perpendicular to the opposite side. (Then use #6, #7)
9. **Intersecting lines** form CONGRUENT vertical angles.
10. When 2 **parallel lines** are crossed by a transversal, (corresponding OR alternate interior) angles are congruent.
11. A **triangle with a right angle** is a right triangle (then use HL)

Angle PAIRS

12. 2 angles that form a **linear pair** are supplementary (then use #13)
13. **Supplements** of congruent angles are congruent.
14. 2 angles that form a **right angle** are complementary (then use #15)
15. **Complements** of congruent angles are congruent.

ISOSCELES TRIANGLES

16. In a triangle, sides opposite **congruent angles** are congruent
17. In a triangle, angles opposite **congruent sides** are congruent.

POSTULATE (assumed to be true without a proof)

Stated for equality, same holds true for congruence.

- a. **Addition Postulate:** When = quantities are added to = quantities, the sums are =.
- b. **Subtraction Postulate:** When = quantities are subtracted from = quantities, the differences are =.
- c. **Multiplication Postulate:** When = quantities are multiplied by = non-zero quantities, the products are =.
Special Case: Doubles of equal quantities are equal.
- d. **Division Postulates:** When = quantities are divided by = non-zero quantities, the quotients are equal.
Special Case: Halves of equal quantities are equal.
- e. **Substitution:** A quantity may be substituted for its equal in any expression.
- f. **Partition:** The whole is equal to the sum of its parts.

PROPERTIES (something that can be proven true)

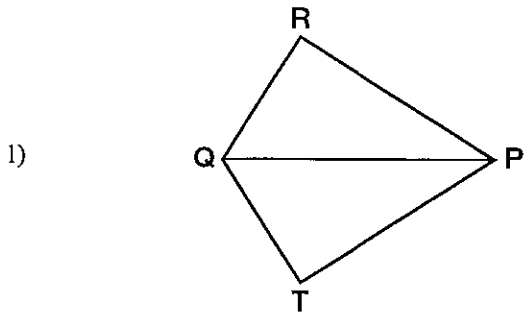
Stated for equality, same holds true for congruence.

- a. **Transitive:** If $a = b$ and $b = c$, then $a = c$
- b. **Symmetric:** If $a = b$, then $b = a$
- c. **Reflexive:** A quantity is = to itself.

7

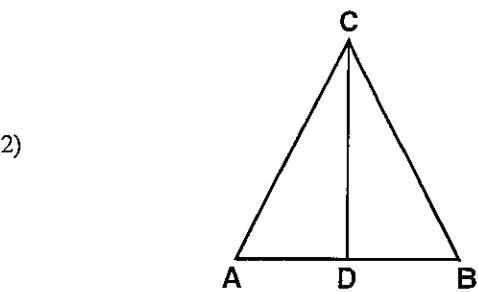
LESSON 3: Two Column Congruent Triangle Proofs

SEGMENTS



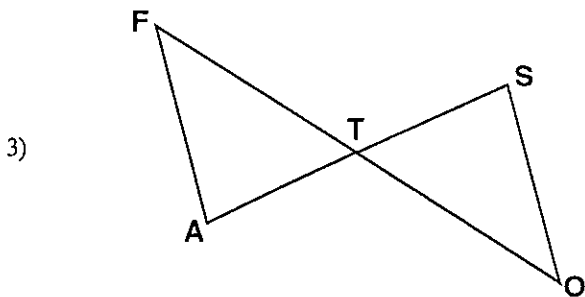
Given: $\overline{QR} \cong \overline{QT}$
 $\angle RQP \cong \angle TQP$

Prove: $\triangle QRP \cong \triangle QTP$



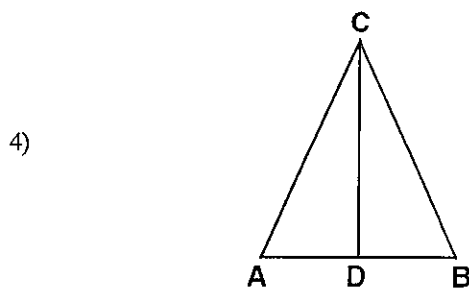
Given: $\overline{AC} \cong \overline{CB}$
 D is the midpoint of \overline{AB}

Prove: $\triangle ACD \cong \triangle BCD$



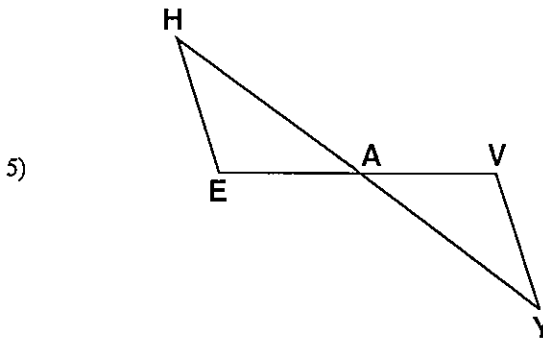
Given: \overline{FO} bisects \overline{AS}
 $\angle A \cong \angle S$

Prove: $\triangle FAT \cong \triangle OST$



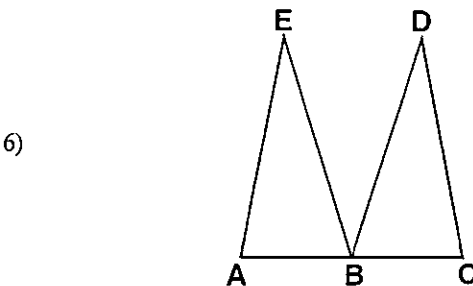
Given: $AC \cong CB$
 D is midpoint of \overline{AB}

Prove: $\angle ACD \cong \angle DCB$



Given: \overline{HY} and \overline{EV} bisect each other

Prove: $\overline{HE} \cong \overline{VY}$

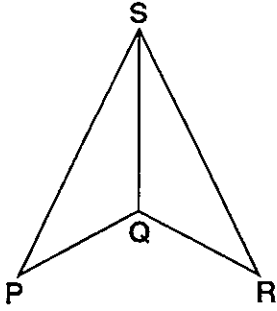


Given: $\angle E \cong \angle D$
 $\angle A \cong \angle C$
 B is the midpoint of \overline{AC}

Prove: $\overline{EA} \cong \overline{DC}$

ANGLES

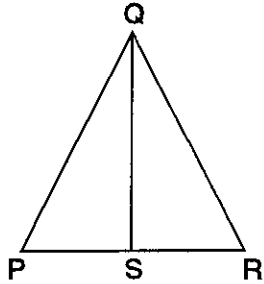
7)



Given: \overline{SQ} bisects $\angle PSR$
 $\angle P \cong \angle R$

Prove: $\triangle PQS \cong \triangle QSR$

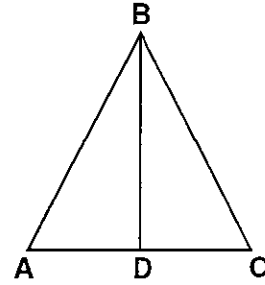
8)



Given: S is the midpoint of \overline{PR}
 \overline{QS} is the altitude from Q to \overline{PR}

Prove: $\triangle PQS \cong \triangle RSQ$

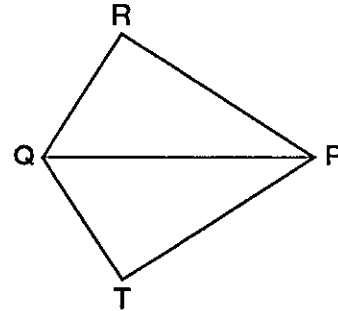
9)



Given: $\overline{BD} \perp \overline{AC}$
 $\overline{AB} \cong \overline{BC}$

Prove: $\triangle ABD \cong \triangle CBD$

10)

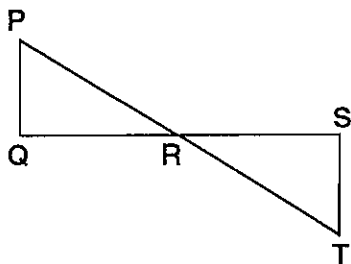


Given: $\overline{QR} \perp \overline{RP}$
 $\overline{QT} \perp \overline{TP}$
 $\overline{QR} \cong \overline{QT}$

Prove: $\triangle QRP \cong \triangle QTP$

SEGMENTS AND ANGLES

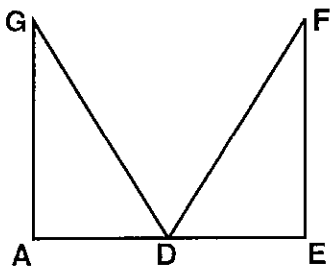
11)



Given: \overline{PT} bisects \overline{QS}
 $\overline{PQ} \perp \overline{QS}$
 $\overline{TS} \perp \overline{QS}$

Prove: $\triangle PQR \cong \triangle RST$

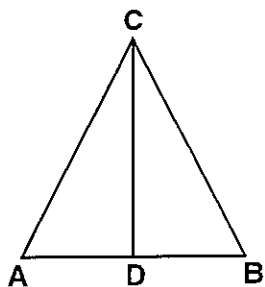
12)



Given: $GA = FE$
 D is the midpoint of \overline{AE}
 $\overline{GA} \perp \overline{AE}$
 $\overline{FE} \perp \overline{AE}$

Prove: $\triangle DAG \cong \triangle DEF$

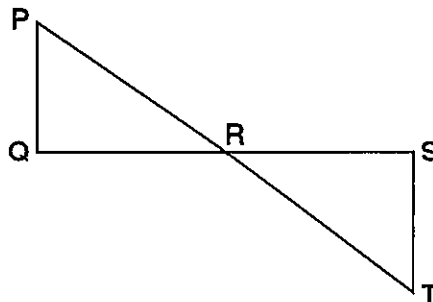
13)



Given: $\overline{CD} \perp \overline{AB}$
 D is the midpoint of \overline{AB}

Prove: $\triangle ADC \cong \triangle BDC$

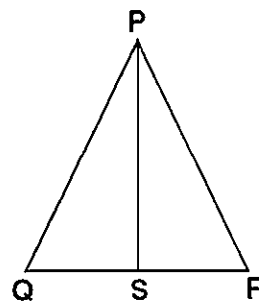
14)



Given: $\overline{PQ} \perp \overline{QS}$
 $\overline{TS} \perp \overline{QS}$
 R is the midpoint of \overline{QS}

Prove: $\angle P \cong \angle T$

15)

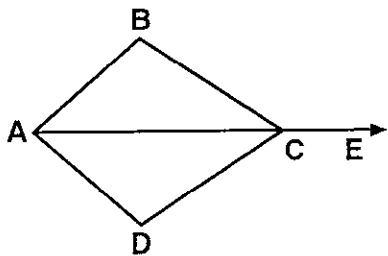


Given: $\overline{PS} \perp \overline{QR}$
 \overline{PS} bisects $\angle QPR$

Prove: $\angle Q \cong \angle R$

ANGLE PAIRS

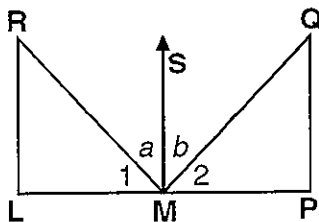
16)



Given: $\angle BCE \cong \angle DCE$
 \overline{AC} bisects $\angle BAD$

Prove: $\triangle ABC \cong \triangle ADC$

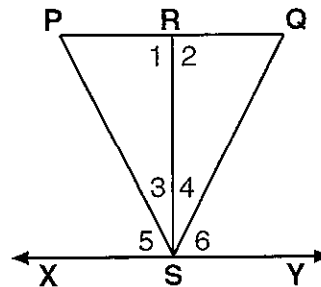
17)



Given: \overline{MS} is \perp bisector of \overline{LP}
 $RM = MQ$
 $\angle a \cong \angle b$

Prove: $\triangle RLM \cong \triangle QPM$

18)

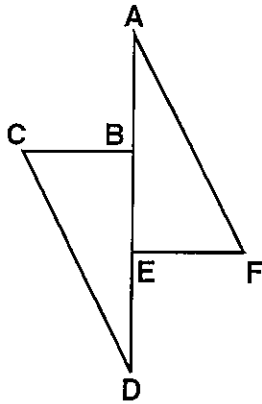


Given: $\overline{RS} \perp \overline{XY}$
 $\angle 5 \cong \angle 6$
 $\angle P \cong \angle Q$
 $\overline{PS} \cong \overline{QS}$

Prove: $\triangle PRS \cong \triangle QRS$

ADDITION AND SUBTRACTION

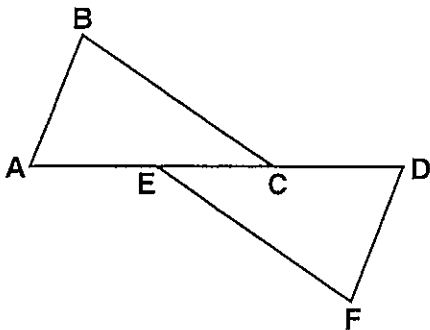
19)



Given: $\overline{AB} \cong \overline{ED}$
 $\overline{FE} \cong \overline{CB}$
 $\overline{FE} \perp \overline{AD}$
 $\overline{CB} \perp \overline{AD}$

Prove: $\triangle AEF \cong \triangle CBD$

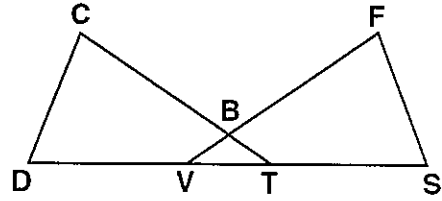
20)



Given: $\angle A \cong \angle D$
 $\overline{AE} \cong \overline{CD}$
 $\angle AEF \cong \angle BCD$

Prove: $\triangle ABC \cong \triangle DFE$

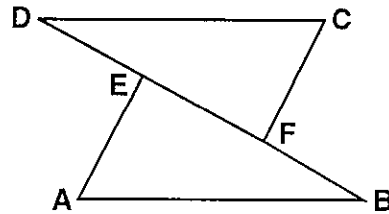
21)



Given: $\overline{CB} \cong \overline{FB}$
 $\overline{BT} \cong \overline{BV}$
 $\overline{DV} \cong \overline{TS}$
 $\overline{DC} \cong \overline{FS}$

Prove: $\angle D \cong \angle S$

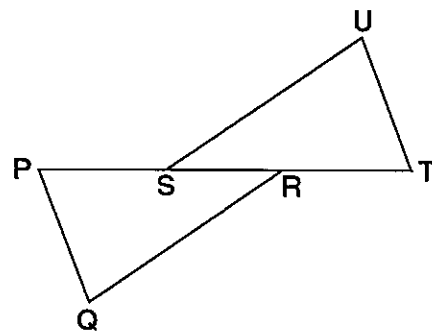
22)



Given: $\overline{AE} \perp \overline{DB}$
 $\overline{CF} \perp \overline{DB}$
 $DE = FB$
 $DC = AB$

Prove: $\triangle ABE \cong \triangle CDF$

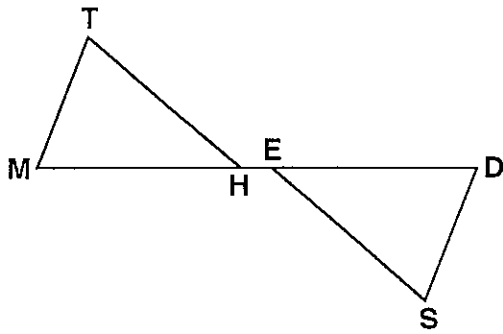
23)



Given: $\overline{SU} \cong \overline{QR}$
 $\overline{PS} \cong \overline{RT}$
 $\angle PSU \cong \angle QRT$

Prove: $\triangle PQR \cong \triangle STU$

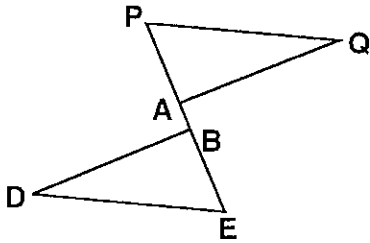
24)



Given: $\angle M \cong \angle D$
 $\overline{ME} \cong \overline{HD}$
 $\angle THE \cong \angle SEH$

Prove: $\triangle TMH \cong \triangle SDE$

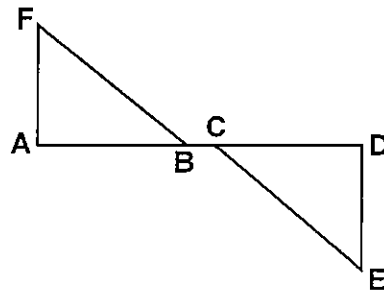
25)



Given: $PQ = DE$
 $PB = AE$
 $\overline{QA} \perp \overline{PE}$
 $\overline{DB} \perp \overline{PE}$

Prove: $\angle D \cong \angle Q$

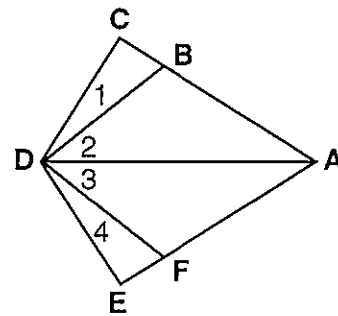
26)



Given: $\overline{FB} \cong \overline{CE}$
 $\overline{AC} \cong \overline{BD}$
 $\overline{FA} \perp \overline{AD}$
 $\overline{DE} \perp \overline{AD}$

Prove: $\overline{AF} \cong \overline{DE}$

27)

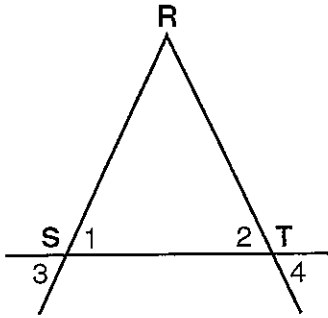


Given: \overline{DA} bisects $\angle BDF$
 $\angle 1 \cong \angle 4$
 $\overline{CD} \cong \overline{DE}$

Prove: $\triangle CDA \cong \triangle EDA$

ISOSCELES TRIANGLES

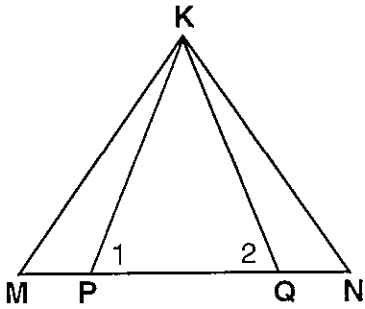
28)



Given: $RS = RT$

Prove: $\angle 3 \cong \angle 4$

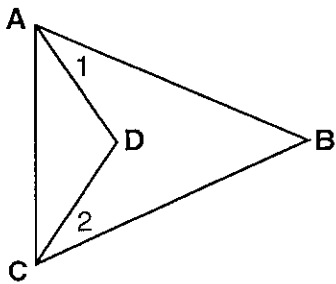
29)



Given: $\overline{MP} \cong \overline{QN}$
 $\angle 1 \cong \angle 2$

Prove: $\triangle MKN$ is isosceles

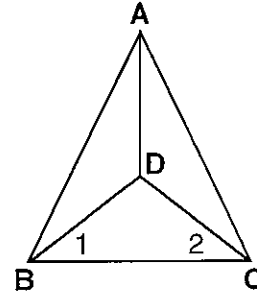
30)



Given: $\overline{AB} \cong \overline{BC}$
 $\angle 1 \cong \angle 2$

Prove: $\triangle ADC$ is isosceles

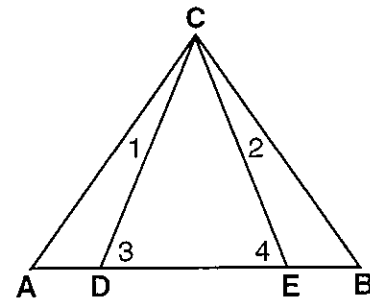
31)



Given: $\overline{AB} \cong \overline{AC}$
 \overline{AD} bisects $\angle BAC$

Prove: $\angle 1 \cong \angle 2$

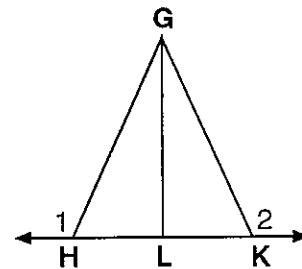
32)



Given: $\overline{CA} \cong \overline{CB}$
 $\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$

33)

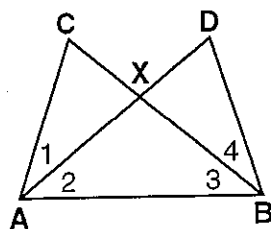


Given: $\angle 1 \cong \angle 2$

Prove: $\overline{GH} \cong \overline{GK}$

OVERLAPPING TRIANGLES

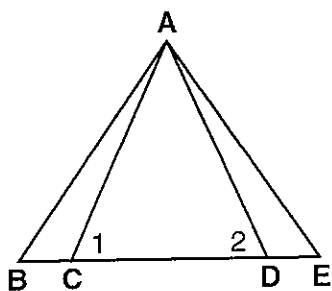
34)



Given: \overline{BC} and \overline{AD} intersect at X
 $\overline{AC} \cong \overline{DB}$
 $\angle CAB \cong \angle DBA$
 $\angle 2 \cong \angle 3$

Prove: $\overline{CX} \cong \overline{XD}$

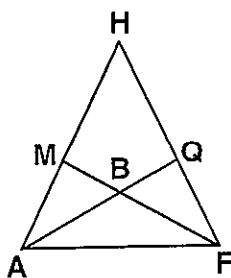
35)



Given: $\overline{BD} \cong \overline{CE}$
 $\overline{AC} \cong \overline{AD}$

Prove: $\angle B \cong \angle E$

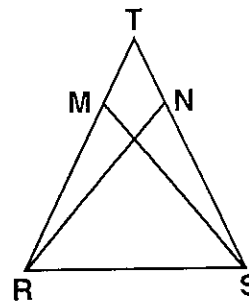
36)



Given: $\overline{HA} \cong \overline{HF}$
 $\overline{HM} \cong \overline{HQ}$

Prove: $\overline{FM} \cong \overline{AQ}$

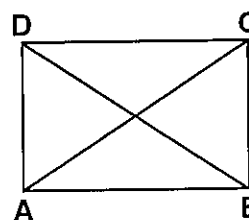
37)



Given: $\overline{TR} \cong \overline{TS}$
 $\overline{MR} \cong \overline{NS}$

Prove: $\overline{RN} \cong \overline{MS}$

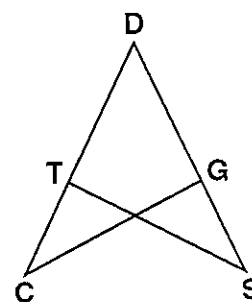
38)



Given: $\overline{DA} \cong \overline{CB}$
 $\overline{DA} \perp \overline{AB}$
 $\overline{CB} \perp \overline{AB}$

Prove: $\overline{BD} \cong \overline{AC}$

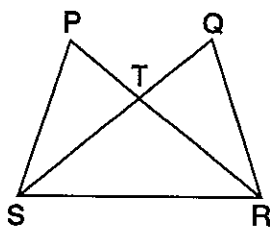
39)



Given: $\overline{DTC} \cong \overline{DGS}$
 $\angle C \cong \angle S$

Prove: $\overline{ST} \cong \overline{CG}$

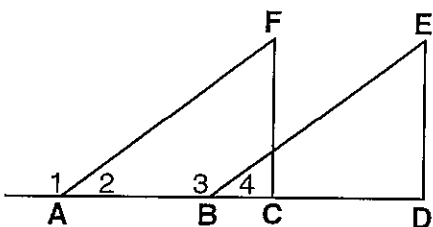
40)



Given: $\overline{TS} \cong \overline{TR}$
 $\angle P \cong \angle Q$

Prove: $\overline{PS} \cong \overline{QR}$

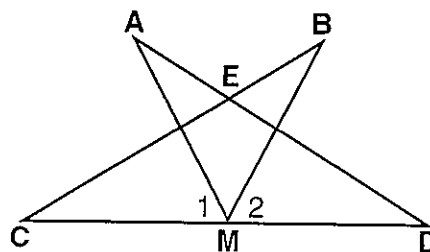
41)



Given: $\angle 2 \cong \angle 4$
 $\overline{AB} \cong \overline{CD}$
 $\overline{FC} \perp \overline{AD}$
 $\overline{ED} \perp \overline{AD}$

Prove: $\overline{FC} \cong \overline{ED}$

42)

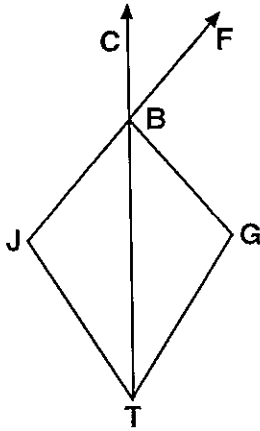


Given: $\angle C \cong \angle D$
 $\angle 1 \cong \angle 2$
 M is the midpoint of \overline{CD}

Prove: $\overline{CB} \cong \overline{DA}$

TRANSITIVE AND SUBSTITUTION

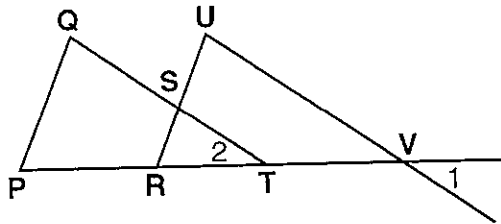
43)



Given: $\angle CBF \cong \angle TBG$
 \overline{TB} bisects $\angle JTG$

Prove: $\triangle BJT \cong \triangle BGT$

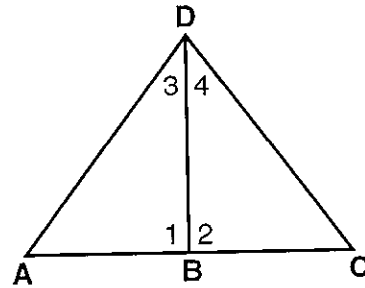
44)



Given: $\overline{PR} \cong \overline{TV}$
 $\angle 1 \cong \angle 2$
 $\overline{QT} \cong \overline{UV}$

Prove: $\angle Q \cong \angle U$

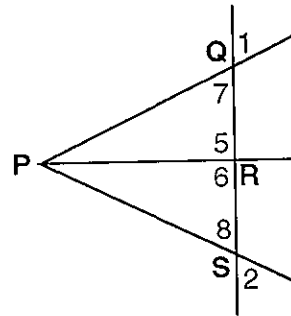
45)



Given: $\angle A \cong \angle C$
 $\angle C \cong \angle 4$
 \overline{DB} bisects $\angle ADC$
 $\overline{AD} \cong \overline{DC}$

Prove: $\triangle ADB \cong \triangle CDB$

46)



Given: $\overline{PQ} \cong \overline{PS}$
 \overline{PR} bisects $\angle QPS$

Prove: $\angle 1 \cong \angle 2$