

Geometry

Unit 1-4

Segments, Angles and Triangles

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Lesson 1: Segments

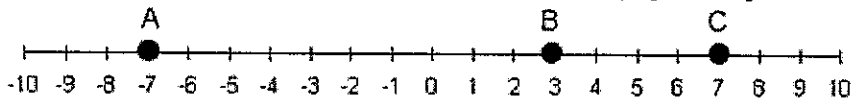
Definition:

A segment (or line segment) is a part of a line that is bounded by two distinct end points. It contains every point on the line between its end points.

Ruler Postulate

Ruler Postulate: The points on a line can be put into a one-to-one correspondence (paired) with the real numbers. The distance between any two points is represented by the absolute value of the difference between the numbers. [Keep in mind that distances are always positive.]

Example 1



a) The distance between A and B is _____

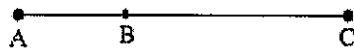
b) The distance between A and C is _____

c) The distance between B and C is _____

Segment Addition Postulate

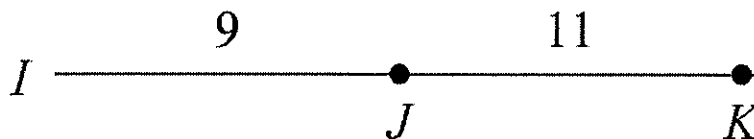
Statement: If B lies on the segment from A to C , then $AB + BC = AC$.

Also the converse: If $AB + BC = AC$, then B lies on the segment from A to C .

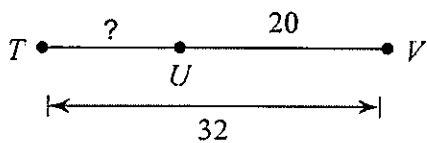


$$AB + BC = AC$$

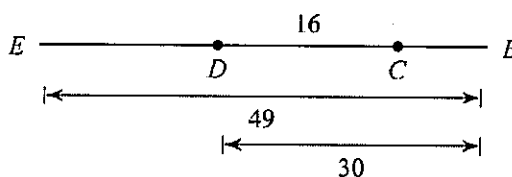
Example 2: Find IK .



Example 3:
Find TU.



Example 4:
Find EC



Example 5:

Points A, B, and C are collinear. Point B is between A and C.

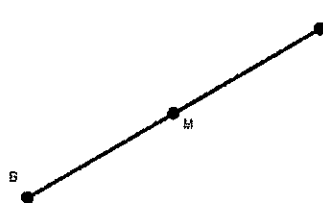
a) Find AC if AB = 16 and BC = 12. b) Find AB if AC = 20 and CB = 6.

c) AC = 22, BC = $x + 14$, and AB = $x + 10$.
Find x.

Midpoint of a Segment

Definition: The midpoint of a segment is a point on the segment forming two congruent segments.

Example 6: If M is the midpoint of \overline{BA} ,
and MA = 6 what is BA?

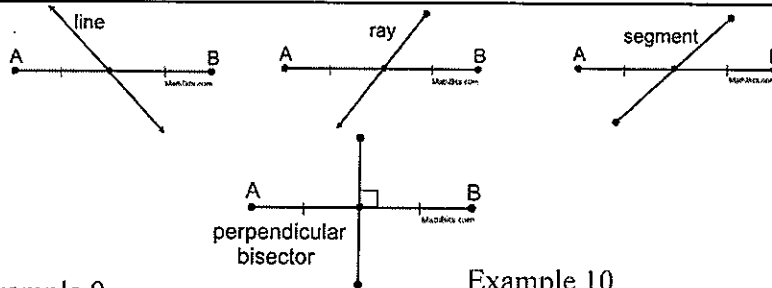


Example 7: A is the midpoint of \overline{CT} . If CA = $2x + 1$ and
AT = $x + 6$, find CT.

Example 8: O is the midpoint of \overline{DG} . If DO = $x + 6$ and
DG = $4x + 11$, find OG.

Bisector of a Segment

Definition: The bisector of a segment is a line, ray, or segment which cuts the given segment into two congruent segments.

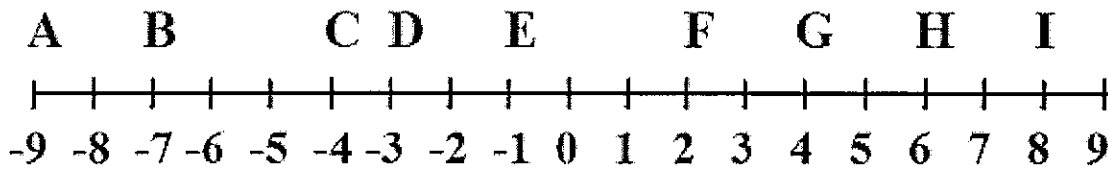


Example 9
 \overline{BR} bisects \overline{AD} at point E.
If $AE = x^2$ and $ED = 2x + 15$,
find the length of \overline{AD} .

Example 10
 \overline{SO} is the perpendicular bisector of \overline{LV} at point E.
If $LE = x^2$ and $ED = x + 20$,
solve for x.

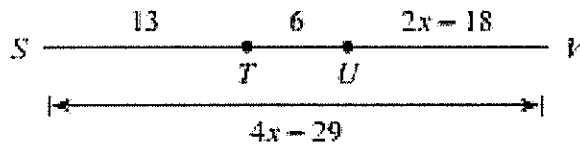
Example 11
 \overline{DA} is the perpendicular bisector of \overline{IS} at point Y.
If $m\angle DYS = x^2 - x$, find the
value of x.

Use the diagram below to answer the questions 1-4.
Show the calculation that you used to obtain your answer.



1. What is the distance between points C and H?
2. What is the distance between points I and A?
3. What is the distance between points G and C?
4. What is the distance between points B and G?

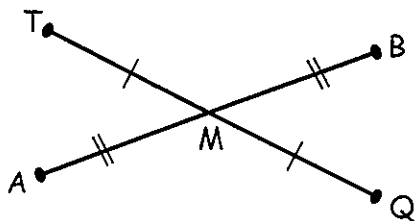
5. Solve for x.



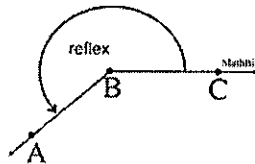
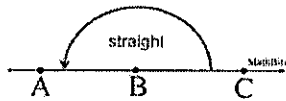
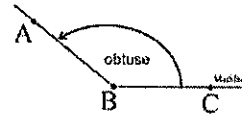
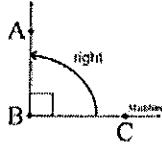
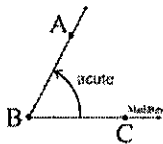
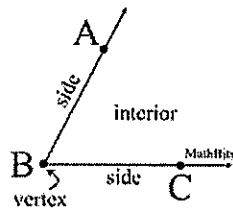
6. A is the midpoint of \overline{CR} . If $CA = x + 4$ and $AR = \frac{2}{3}x + 5$, find the length of \overline{CR} .

7. E is the midpoint of \overline{DN} . If $DN = x^2$ and $DE = x + 24$. Solve for the positive value of x.

8. If $AM = 5y - 1$, $AB = 38$, find the value of y.



Lesson 2: Angles and Angle Pairs



Pairs of Angles

- Adjacent Angles are two angles that share a common vertex, a common side, and no common interior points. (They share a vertex and side, but do not overlap.)

Draw a picture

- A Linear Pair is two **adjacent** angles whose non-common sides form opposite rays.

Draw a picture

- Complementary Angles are two angles the sum of whose measures is 90° .
Draw a picture

- Supplementary Angles are two angles the sum of whose measures is 180° .
Draw a picture

Working With Angles

Angle Addition Postulate

If D lies in the interior of $\angle ABC$, then
 $m\angle ABD + m\angle DBC = m\angle ABC$.

Draw a picture

Angle Bisector

Definition: An angle bisector is a ray from the vertex of the angle into the interior of the angle forming two congruent angles.

Draw a picture

Angles Forming a Straight Line

If the non-shared sides of two, or more, adjacent angles form a straight line, the measures of the angles add up to 180° .

Draw a picture

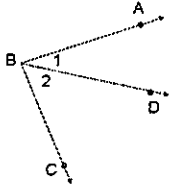
Angles Around a Point

If two, or more, adjacent angles completely surround a point, the measures of the angles add up to 360° .

Draw a picture

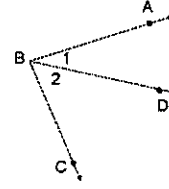
Example 1:

If the $m\angle 1 = 35^\circ$ and the $m\angle 2 = 40^\circ$, find the $m\angle ABC$.



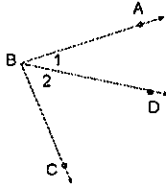
Example 2:

If the $m\angle 1 = 20^\circ$ and the $m\angle ABC = 65^\circ$, find $m\angle 2$.



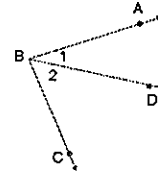
Example 3:

If the $m\angle 1 = (3x+5)^\circ$, the $m\angle 2 = (x+10)^\circ$ and the $m\angle ABC = 95^\circ$, find the $m\angle 2$.



Example 4:

If $m\angle 1:m\angle 2$ is 2:3 and the $m\angle ABC$ is 45° , find the $m\angle 1$.

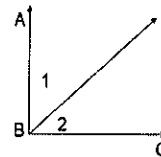


Example 5:

$\angle ABC$ is a right angle. If $m\angle ABC = (x^2 - 43x)^\circ$, find the positive value of x .

Example 6:

If $AB \perp BC$ and $m\angle 1 = 2x + 40$ and $m\angle 2 = 4x - 10$, find the measure of both angles.



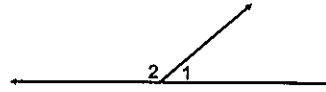
Example 7:

a) What is the complement of an angle that measures 30° ?

b) An angle measures 15 more than twice its complement.
Find both angles.

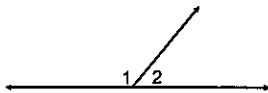
Example 8:

If the $m\angle 1 = 45^\circ$, find $m\angle 2$.



Example 9:

If the $m\angle 1 = (2x + 40)^\circ$ and the $m\angle 2 = (3x - 10)^\circ$, find $m\angle 1$.



Example 10:

Find the supplement of a 73° angle.

Example 11:

Two angles are supplementary. The measure of the larger angle is twice the measure of the smaller angle.
Find the measure of the larger angle.

Example 12:

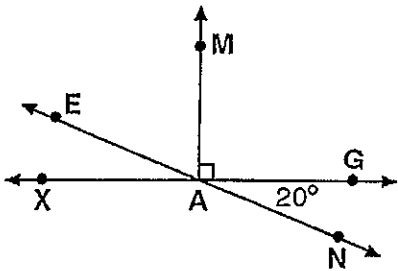
1. The complement of $3x - 20$ is _____.
2. The supplement of $20 - 8x$ is _____.

___ 12) Is 145° an acute angle, right angle, obtuse angle, or straight angle?

___ 13) Is 180° an acute angle, right angle, obtuse angle, or straight angle?

Questions 14 and 15 refer to the following:

In the diagram below, \overline{XG} and \overline{EN} intersect at A, $\overline{AM} \perp \overline{XG}$, and $m\angle GAN = 20^\circ$.



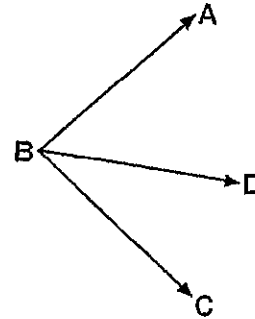
___ 14) Name an angle adjacent to $\angle MAG$.

___ 15) Find the measure of the supplement of $\angle EAG$.

___ 16) $\angle 1$ and $\angle 2$ are supplementary. If $m\angle 1 = (3x - 17)^\circ$ and $m\angle 2 = (5x + 21)^\circ$, find the value of x .

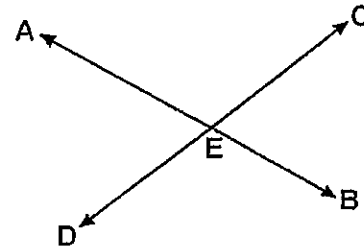
___ 17) $\angle 1$ and $\angle 2$ are complementary. If $m\angle 1 = (x + 3)^\circ$ and $m\angle 2 = (4x - 8)^\circ$, find the value of x .

___ 18) In the accompanying diagram, $\overline{BA} \perp \overline{BC}$, and \overline{BD} is drawn.



If $m\angle ABD = (2x + 18)^\circ$ and $m\angle CBD = (4x - 18)^\circ$, find x .

___ 19) In the accompanying diagram, \overline{AB} and \overline{CD} intersect at E.

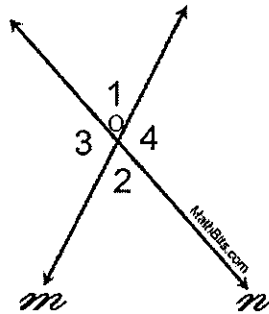


If $m\angle AEC = (2x + 40)^\circ$ and $m\angle CEB = (x + 20)^\circ$, find x .

Lesson 3

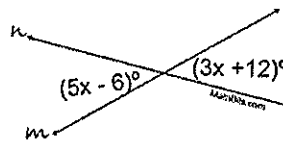
Intersecting Lines and Parallel lines Crossed by a Transversal

Definition: Vertical angles are a pair of non-adjacent angles formed by the intersection of two straight line.



Example 1

Given:
straight lines m and n

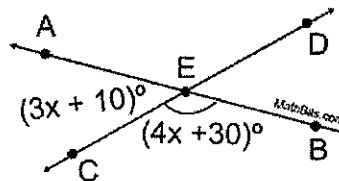


Find the number of degrees
in the indicated angles.

Example 2

Given:
 \overline{AB} and \overline{CD}

Find: $m\angle DEB$



Example 3

Given:

\overline{CD} bisects $\angle ACB$

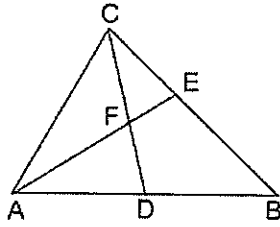
\overline{AE} bisects $\angle CAB$

$m\angle ACB = 48^\circ$

$m\angle CAB = 56^\circ$

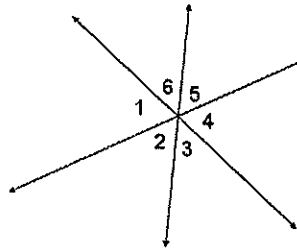
Find: $m\angle CFA$; $m\angle EFD$

$m\angle AFD$; $m\angle CFE$

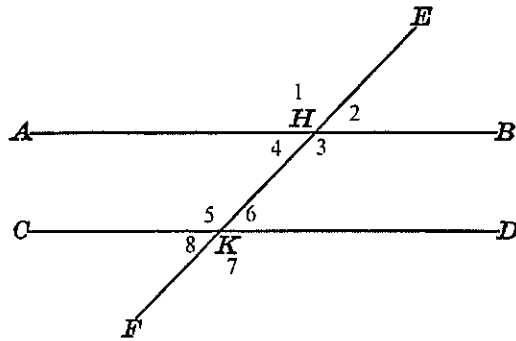


Example 4

If $m\angle 1 = 114^\circ$ and $m\angle 6 = 36^\circ$,
find the measure of all other angles.

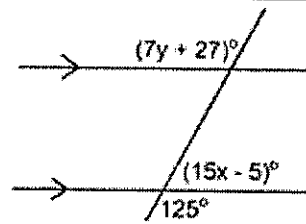


Parallel Lines Crossed by a Transversal



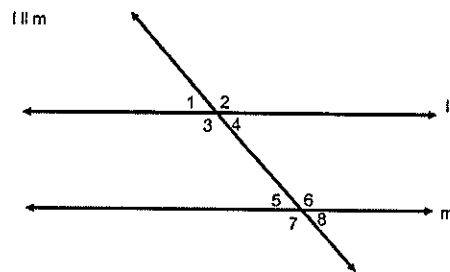
Vertical Angles	
Linear Pairs	
Corresponding Angles	
Alternate Interior Angles	
Alternate Exterior Angles	
Same Side Interior Angles	
Same Side Exterior Angles	

Example 5 Solve for x and y



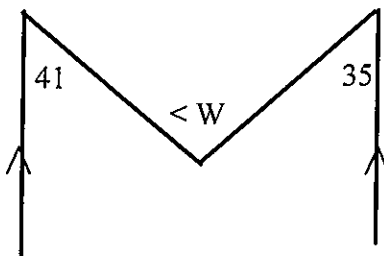
Example 6:

If $m \angle 1 = (5n)^\circ$ and $m \angle 8 = (2n + 66)^\circ$, find n.



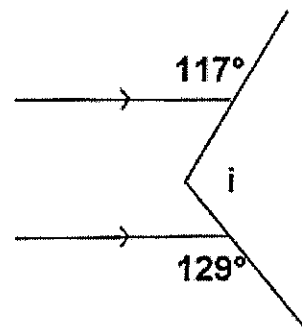
Example 7

Find the measure of $\angle W$.



Example 8

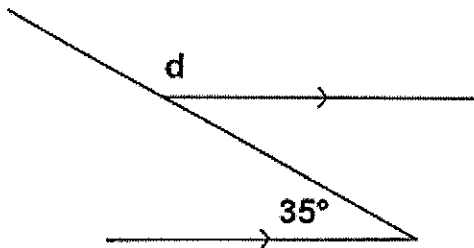
Find the measure of $\angle i$.



In geometry, it may be necessary to add a line or segment to a diagram to help in solving a problem or proving a concept. Such an added line or segment is called an auxiliary line. The word "auxiliary" means providing additional help or support.

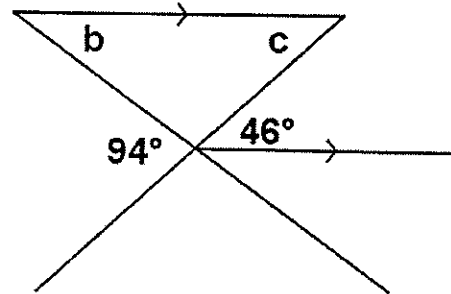
Example 9

Find the measure of $\angle d$.



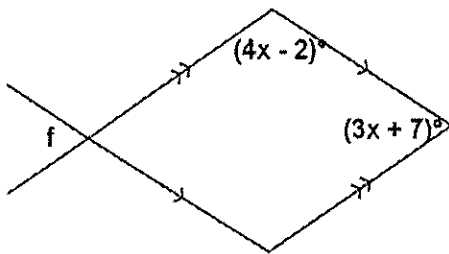
Example 10

Find the measure of $\angle b$ and $\angle c$.



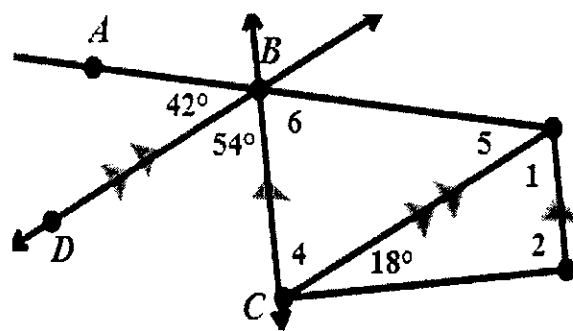
Example 11

Solve for x .



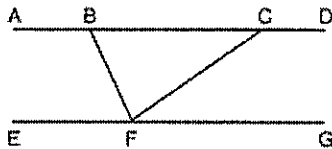
Example 12

Find the measure of all numbered angles



Example 13

Steve drew line segments \overline{ABCD} , \overline{EFG} , \overline{BF} , and \overline{CF} as shown in the diagram below. Scalene $\triangle BFC$ is formed.

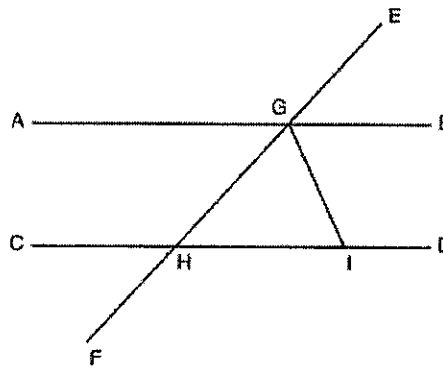


Which statement will allow Steve to prove $\overline{ABCD} \parallel \overline{EFG}$?

- 1 $\angle CFG \cong \angle FCB$
- 2 $\angle ABF \cong \angle BFC$
- 3 $\angle EFB \cong \angle CFB$
- 4 $\angle CBF \cong \angle GFC$

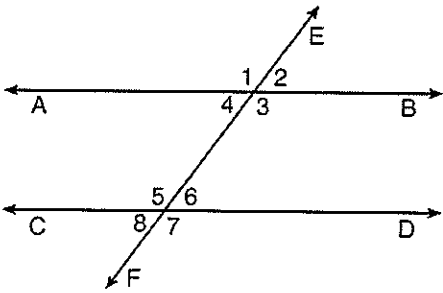
Example 14

In the diagram below, \overline{EF} intersects \overline{AB} and \overline{CD} at G and H , respectively, and \overline{GI} is drawn such that $\overline{GH} \cong \overline{IH}$.



If $m\angle EGB = 50^\circ$ and $m\angle DIG = 115^\circ$, explain why $\overline{AB} \parallel \overline{CD}$.

1 Transversal \overleftrightarrow{EF} intersects \overleftrightarrow{AB} and \overleftrightarrow{CD} , as shown in the diagram below.



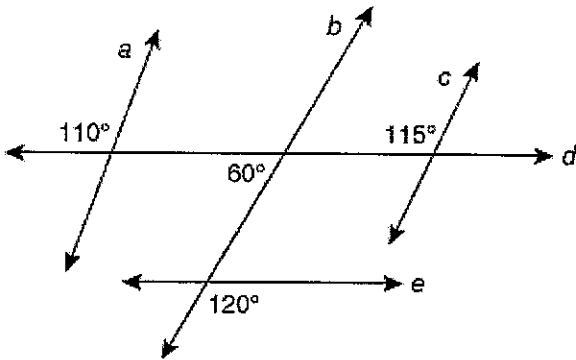
Which statement could always be used to prove $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$?

- 1) $\angle 2 \cong \angle 4$
- 2) $\angle 7 \cong \angle 8$
- 3) $\angle 3$ and $\angle 6$ are supplementary
- 4) $\angle 1$ and $\angle 5$ are supplementary

2 A transversal intersects two lines. Which condition would always make the two lines parallel?

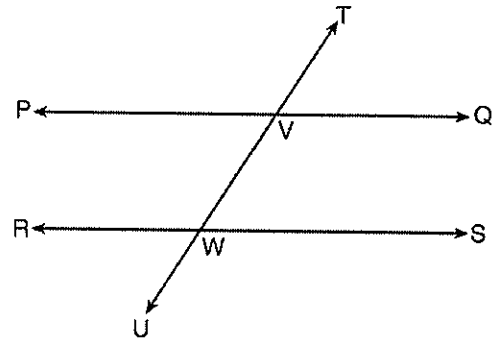
- 1) Vertical angles are congruent.
- 2) Alternate interior angles are congruent.
- 3) Corresponding angles are supplementary.
- 4) Same-side interior angles are complementary.

3 Based on the diagram below, which statement is true?



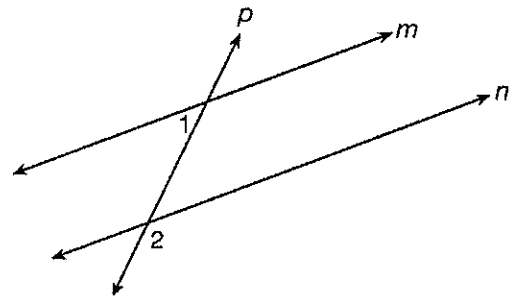
- 1) $a \parallel b$
- 2) $a \parallel c$
- 3) $b \parallel c$
- 4) $d \parallel e$

4 In the diagram below, transversal \overleftrightarrow{TU} intersects \overleftrightarrow{PQ} and \overleftrightarrow{RS} at V and W , respectively.



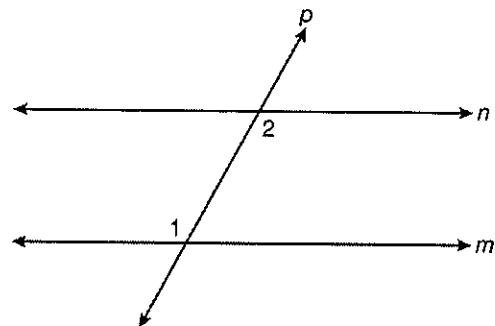
If $m\angle TVQ = 5x - 22$ and $m\angle VWS = 3x + 10$, for which value of x is $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$?

5 As shown in the diagram below, lines m and n are cut by transversal p .



If $m\angle 1 = 4x + 14$ and $m\angle 2 = 8x + 10$, lines m and n are parallel when x equals what number?

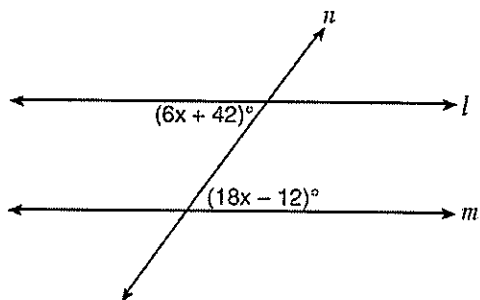
6 In the diagram below, line p intersects line m and line n .



If $m\angle 1 = 7x$ and $m\angle 2 = 5x + 30$, lines m and n are parallel when x equals

- 1) 12.5
- 2) 15
- 3) 87.5
- 4) 105

- 7 Line n intersects lines l and m , forming the angles shown in the diagram below.

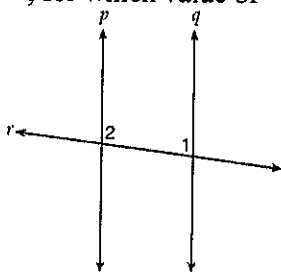


Which value of x would prove $l \parallel m$?

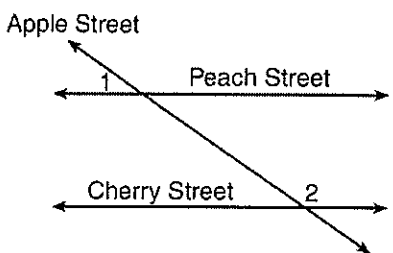
- 1) 2.5
- 2) 4.5
- 3) 6.25
- 4) 8.75

- 8 Lines p and q are intersected by line r , as shown below. If $m\angle 1 = 7x - 36$ and $m\angle 2 = 5x + 12$, for which value of x would $p \parallel q$?

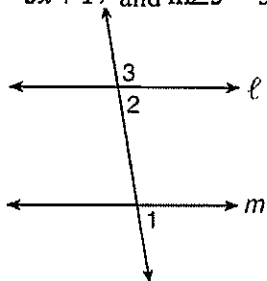
- 1) 17
- 2) 24
- 3) 83
- 4) 97



- 9 Peach Street and Cherry Street are parallel. Apple Street intersects them, as shown in the diagram below. If $m\angle 1 = 2x + 36$ and $m\angle 2 = 7x - 9$, what is $m\angle 1$?

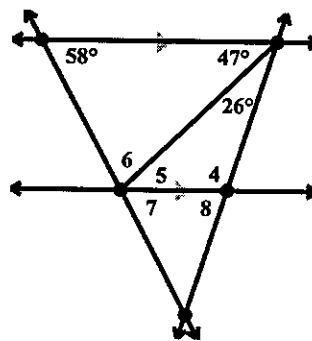


- 10 In the diagram below, line ℓ is parallel to line m , and line w is a transversal. If $m\angle 2 = 3x + 17$ and $m\angle 3 = 5x - 21$, what is $m\angle 1$?

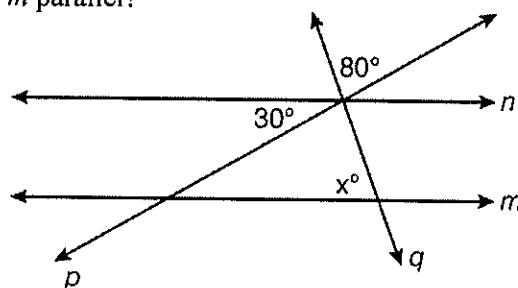


(Not drawn to scale)

11. Determine the measures of all numbered angles

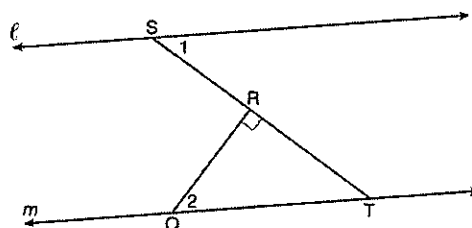


- 12 In the diagram below, lines n and m are cut by transversals p and q . What value of x would make lines n and m parallel?



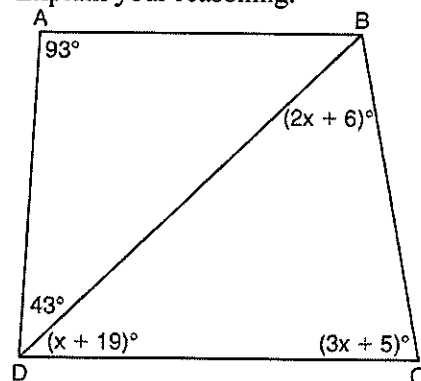
- 1) 110
- 2) 80
- 3) 70
- 4) 50

- 13 In the diagram below, $\ell \parallel m$ and $\overline{QR} \perp \overline{ST}$ at R .



If $m\angle 1 = 63$, find $m\angle 2$.

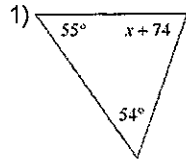
- 14 In the diagram below of quadrilateral $ABCD$ with diagonal \overline{BD} , $m\angle A = 93$, $m\angle ADB = 43$, $m\angle C = 3x + 5$, $m\angle BDC = x + 19$, and $m\angle DBC = 2x + 6$. Determine if \overline{AB} is parallel to \overline{DC} . Explain your reasoning.



Lesson 4: Triangle Properties

THEOREM: The sum of the measures of the interior angles of a triangle equals 180° .

Solve for x .



2) 

Find the measure of angle A.

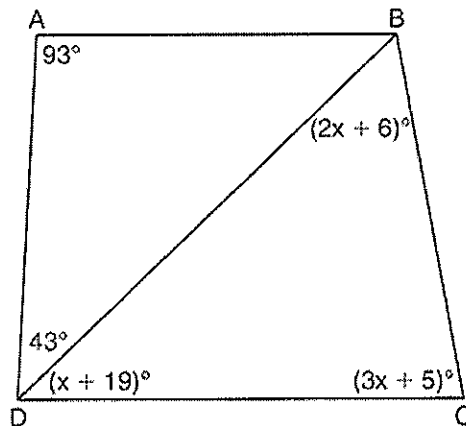
3) 

4) 

Example 5:

In the diagram below of quadrilateral $ABCD$ with diagonal \overline{BD} , $m\angle A = 93$, $m\angle ADB = 43$, $m\angle C = 3x + 5$, $m\angle BDC = x + 19$ and $m\angle DBC = 2x + 6$.

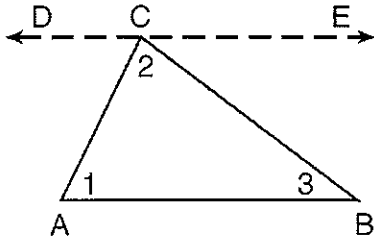
Determine if \overline{AB} is parallel to \overline{DC} . Explain your reasoning.



Example 6

Prove the theorem,

“The sum of the measures of the interior angles of a triangle is 180° .”

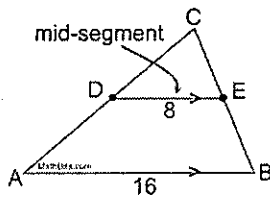


Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$

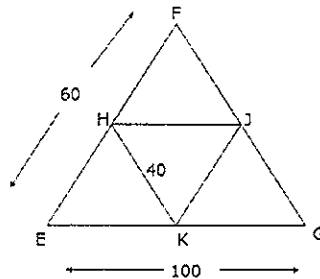
THEOREM:

“Mid-Segment Theorem”: The mid-segment of a triangle, which joins the midpoints of two sides of a triangle, is parallel to the third side of the triangle and half the length of that third side of the triangle.



Example 7

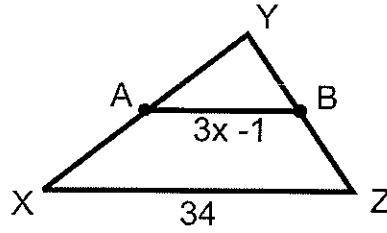
In $\triangle EFG$, H, J, and K are midpoints. Find HJ, JK, and FG.



- HJ:
- JK:
- FG:

Example 8

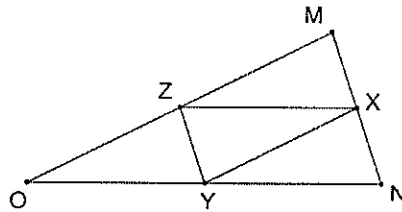
If A and B are midpoints of \overline{XY} and \overline{YZ} , solve for x:



Example 9

Given Z, X, and Y are midpoints of \overline{OM} , \overline{MN} , and \overline{ON} .

If $YZ = 2x + 3$, and $MN = 5x - 14$, then $YZ =$ _____



Date _____

Lesson 4 Triangle Properties HW

Geometry CC

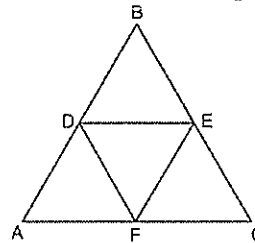
- The angles of triangle ABC are in the ratio of 8:3:4. What is the measure of the *smallest* angle?
 - 12°
 - 24°
 - 36°
 - 72°
- In $\triangle ABC$, $m\angle A = 3x + 1$, $m\angle B = 4x - 17$, and $m\angle C = 5x - 20$. Which type of triangle is $\triangle ABC$?
 - right
 - scalene
 - isosceles
 - equilateral
- Juliann plans on drawing $\triangle ABC$, where the measure of $\angle A$ can range from 50° to 60° and the measure of $\angle B$ can range from 90° to 100° . Given these conditions, what is the correct range of measures possible for $\angle C$?
 - 20° to 40°
 - 30° to 50°
 - 80° to 90°
 - 120° to 130°
- In right $\triangle DEF$, $m\angle D = 90$ and $m\angle F$ is 12 degrees less than twice $m\angle E$. Find $m\angle E$.

- In $\triangle ABC$, the measure of angle A is fifteen less than twice the measure of angle B . The measure of angle C equals the sum of the measures of angle A and angle B . Determine the measure of angle B .

- In the diagram below, the vertices of $\triangle DEF$ are the midpoints of the sides of equilateral triangle ABC , and the perimeter of $\triangle ABC$ is 36 cm.

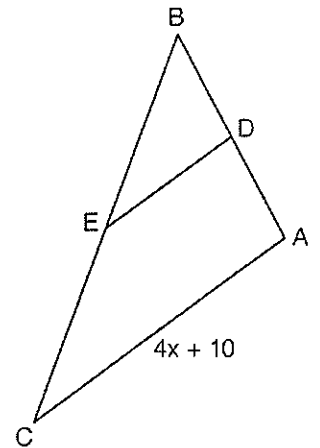
What is the length, in centimeters, of \overline{EF} ?

- 6
- 12
- 18
- 4



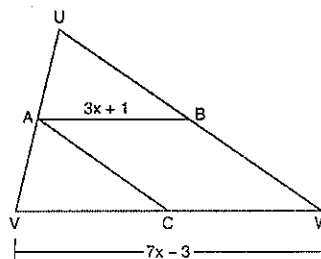
- In the diagram of $\triangle ABC$, D is the midpoint of \overline{AB} , and E is the midpoint of \overline{BC} . If $AC = 4x + 10$, which expression represents DE ?

- $x + 2.5$
- $2x + 5$
- $2x + 10$
- $8x + 20$

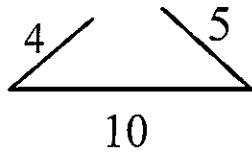


- In the diagram of $\triangle UVW$ below, A is the midpoint of \overline{UV} , B is the midpoint of \overline{UW} , C is the midpoint of \overline{VW} , and \overline{AB} and \overline{AC} are drawn.

If $VW = 7x - 3$ and $AB = 3x + 1$, what is the length of \overline{VC} ?



Lesson 5 Triangle Inequalities



THEOREM: The sum of the lengths of any two sides of a triangle must be greater than the third side.

Example 1

Which sets of numbers could represent the lengths of the sides of a triangle?

{4, 5, 6}

{5, 12, 13}

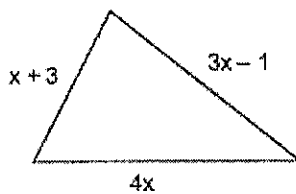
{8, 9, 17}

{2, 10, 13, }

{7, 7, 14}

Example 2

The plot of land illustrated in the accompanying diagram has a perimeter of 34 yards. Find the length, in yards, of *each* side of the figure. Could these measures actually represent the measures of the sides of a triangle? Explain your answer.



Determine the possible lengths of 3rd side of a TRIANGLE

5, 7, _____

The third side of a triangle must be...

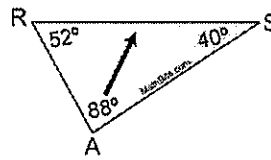
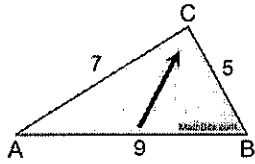
- Less than the sum of the other two sides
 - AND
 - Greater than the difference of the other two sides
-

Examples:

3. If two sides of a triangle have length 8 and 4, what are possible lengths of the 3rd side?

4. If two sides of a triangle have length 3 and 12, what are possible lengths of the 3rd side?

THEOREM: In a triangle, the longest side is across from the largest angle.
Converse: In a triangle, the largest angle is across from the longest side.



Example 5

In $\triangle ABC$, $m\angle A = 95$, $m\angle B = 50$, and $m\angle C = 35$. Which expression correctly relates the lengths of the sides of this triangle?

- 1) $AB < BC < CA$
- 2) $AB < AC < BC$
- 3) $AC < BC < AB$
- 4) $BC < AC < AB$

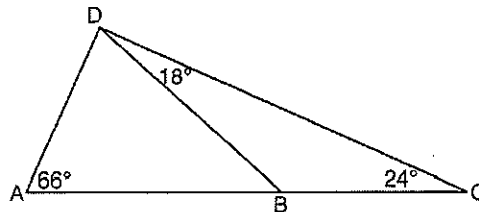
Example 6

In $\triangle ABC$, $AB = 7$, $BC = 8$, and $AC = 9$. Which list has the angles of $\triangle ABC$ in order from smallest to largest?

- 1) $\angle A, \angle B, \angle C$
- 2) $\angle B, \angle A, \angle C$
- 3) $\angle C, \angle B, \angle A$
- 4) $\angle C, \angle A, \angle B$

Example 7

As shown in the diagram of $\triangle ACD$ below, B is a point on \overline{AC} and \overline{DB} is drawn.



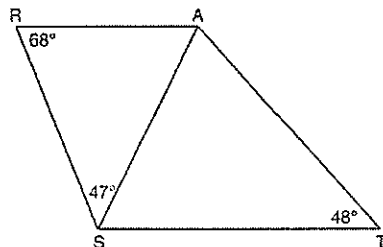
If $m\angle A = 66$, $m\angle CDB = 18$, and $m\angle C = 24$, what is the longest side of $\triangle ABD$?

Example 8

In $\triangle ABC$, $m\angle A = x^2 + 12$, $m\angle B = 11x + 5$, and $m\angle C = 13x - 17$. Determine the longest side of $\triangle ABC$.

Example 9

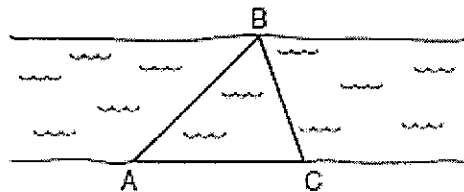
As shown in the diagram below, \overline{AS} is a diagonal of trapezoid $STAR$, $\overline{RA} \parallel \overline{ST}$, $m\angle ATS = 48$, $m\angle RSA = 47$, and $m\angle ARS = 68$.



Determine and state the longest side of $\triangle SAT$.

1. On the banks of a river, surveyors marked locations A , B , and C . The measure of $\angle ACB = 70^\circ$ and the measure of $\angle ABC = 65^\circ$. Which expression shows the relationship between the lengths of the sides of this triangle?

- 1) $AB < BC < AC$
- 2) $BC < AB < AC$
- 3) $BC < AC < AB$
- 4) $AC < AB < BC$



2. In scalene triangle ABC , $m\angle B = 45$ and $m\angle C = 55$. What is the order of the sides in length, from longest to shortest?

- 1) $\overline{AB}, \overline{BC}, \overline{AC}$
- 2) $\overline{BC}, \overline{AC}, \overline{AB}$
- 3) $\overline{AC}, \overline{BC}, \overline{AB}$
- 4) $\overline{BC}, \overline{AB}, \overline{AC}$

3. In $\triangle ABC$, $m\angle A = 65$ and $m\angle B$ is greater than $m\angle A$. The lengths of the sides of $\triangle ABC$ in order from smallest to largest are

- 1) $\overline{AB}, \overline{BC}, \overline{AC}$
- 2) $\overline{BC}, \overline{AB}, \overline{AC}$
- 3) $\overline{AC}, \overline{BC}, \overline{AB}$
- 4) $\overline{AB}, \overline{AC}, \overline{BC}$

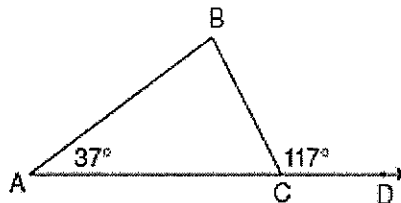
4. In $\triangle PQR$, $PQ = 8$, $QR = 12$, and $RP = 13$. Which statement about the angles of $\triangle PQR$ must be true?

- 1) $m\angle Q > m\angle P > m\angle R$
- 2) $m\angle Q > m\angle R > m\angle P$
- 3) $m\angle R > m\angle P > m\angle Q$
- 4) $m\angle P > m\angle R > m\angle Q$

5. For which measures of the sides of $\triangle ABC$ is angle B the largest angle of the triangle?

- 1) $AB = 2, BC = 6, AC = 7$
- 2) $AB = 6, BC = 12, AC = 8$
- 3) $AB = 16, BC = 9, AC = 10$
- 4) $AB = 18, BC = 14, AC = 5$

6. In the diagram below of $\triangle ABC$ with side \overline{AC} extended through D , $m\angle A = 37$ and $m\angle BCD = 117$. Which side of $\triangle ABC$ is the longest side? Justify your answer.

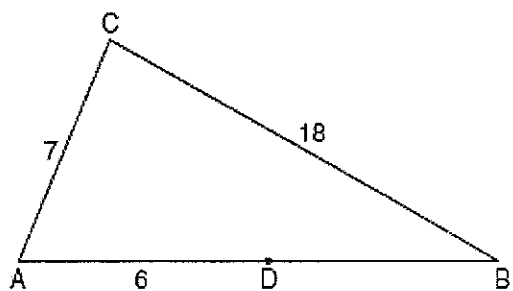


(Not drawn to scale)

7. Which set of numbers represents the lengths of the sides of a triangle?
- 1) {5, 18, 13}
 - 2) {6, 17, 22}
 - 3) {16, 24, 7}
 - 4) {26, 8, 15}
8. Phil is cutting a triangular piece of tile. If the triangle is scalene, which set of numbers could represent the lengths of the sides?
- 1) {2, 4, 7}
 - 2) {4, 5, 6}
 - 3) {3, 5, 8}
 - 4) {5, 5, 8}
9. Which set can *not* represent the lengths of the sides of a triangle?
- 1) {4, 5, 6}
 - 2) {5, 5, 11}
 - 3) {7, 7, 12}
 - 4) {8, 8, 8}
10. In $\triangle ABC$, $AB = 5$ feet and $BC = 3$ feet. Which inequality represents all possible values for the length of \overline{AC} , in feet?
- 1) $2 \leq AC \leq 8$
 - 2) $2 < AC < 8$
 - 3) $3 \leq AC \leq 7$
 - 4) $3 < AC < 7$
11. The lengths of two sides of a triangle are 7 and 11. Which inequality represents all possible values for x , the length of the third side of the triangle?
- 1) $4 \leq x \leq 18$
 - 2) $4 < x \leq 18$
 - 3) $4 \leq x < 18$
 - 4) $4 < x < 18$
12. If two sides of a triangle are 1 and 3, the third side may be
- 1) 5
 - 2) 2
 - 3) 3
 - 4) 4
13. José wants to build a triangular pen for his pet rabbit. He has three lengths of boards already cut that measure 7 feet, 8 feet, and 16 feet. Explain why José cannot construct a pen in the shape of a triangle with sides of 7 feet, 8 feet, and 16 feet.

14. In the diagram below of $\triangle ABC$, D is a point on \overline{AB} , $AC = 7$, $AD = 6$, and $BC = 18$. The length of \overline{DB} could be

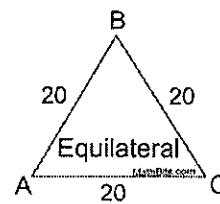
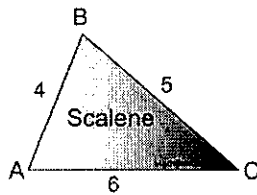
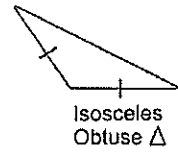
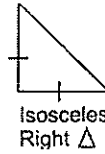
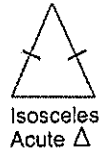
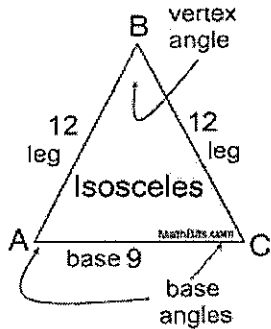
- 1) 5
- 2) 12
- 3) 19
- 4) 25



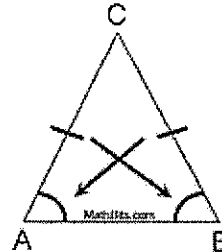
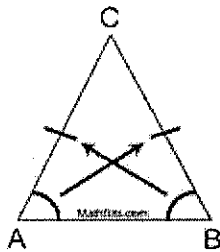
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Lesson 7 Isosceles Triangles

Definition: An isosceles triangle is a triangle with two congruent sides.

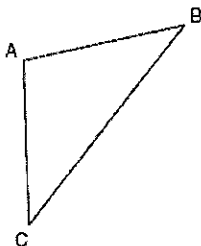


Theorem: In a triangle, sides opposite congruent angles are congruent.
In a triangle, angles opposite congruent sides are congruent.



Examples:

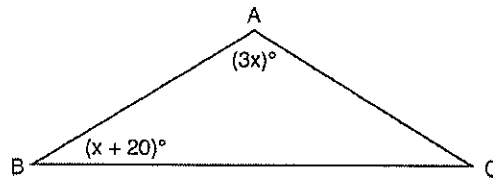
1. In the diagram of ΔABC , $\overline{AB} \cong \overline{AC}$, if $m\angle B = 40^\circ$, what is $m\angle A$?



2. Given ΔJKL with $\overline{JL} \cong \overline{KL}$. If $m\angle J = 58^\circ$, what is $m\angle L$?

Example 3:

In the diagram below of $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $m\angle A = 3x$, and $m\angle B = x + 20$.



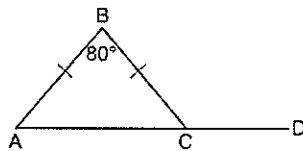
What is the value of x ?

Example 4:

The vertex angle of an isosceles triangle measures 15 degrees more than one of its base angles. How many degrees are there in a base angle of the triangle?

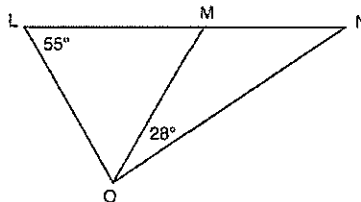
Example 5:

In the diagram below of isosceles $\triangle ABC$, the measure of vertex angle B is 80° . If \overline{AC} extends to point D , what is $m\angle BCD$?



Example 6:

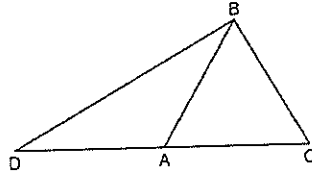
In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$.



If $m\angle L = 55$ and $m\angle NOM = 28$, what is $m\angle N$?

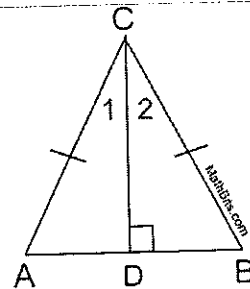
Example 7:

In the diagram of $\triangle BCD$ shown below, \overline{BA} is drawn from vertex B to point A on \overline{DC} , such that $\overline{BC} \cong \overline{BA}$.



In $\triangle DAB$, $m\angle D = x$, $m\angle DAB = 5x - 30$, and $m\angle DBA = 3x - 60$. In $\triangle ABC$, $AB = 6y - 8$ and $BC = 4y - 2$. [Only algebraic solutions can receive full credit.] Find $m\angle D$. Find $m\angle BAC$. Find the length of \overline{BC} . Find the length of \overline{DC} .

Altitudes in Isosceles Triangles



THEOREM: The altitude to the base of an isosceles triangle bisects the vertex angle.

THEOREM: The altitude to the base of an isosceles triangle bisects the base.

Example 8:

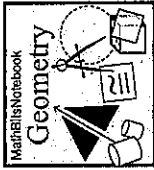
In $\triangle ABC$, $\overline{AB} \cong \overline{BC}$. An altitude is drawn from B to \overline{AC} and intersects \overline{AC} at D . Which conclusion is *not* always true?

- 1) $\angle ABD \cong \angle CBD$
- 2) $\angle BDA \cong \angle BDC$
- 3) $\overline{AD} \cong \overline{BD}$
- 4) $\overline{AD} \cong \overline{DC}$

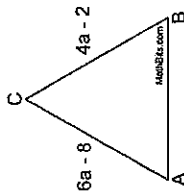
Isosceles Triangles Practice

Name _____

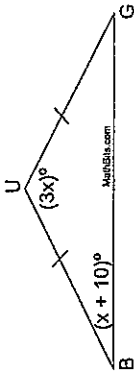
Directions: Read carefully! Please show your work.



1. $\triangle ABC$ is isosceles where $AC = CB$.
 $AC = 6a - 8$ and $CB = 4a - 2$
 Find AC .

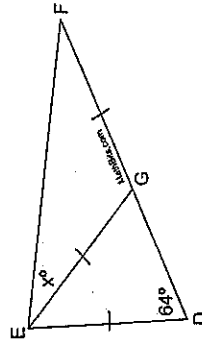


2. $\triangle BUG$ is isosceles.
 $m\angle B = x + 10$ and $m\angle U = 3x$
 Find $m\angle U$.

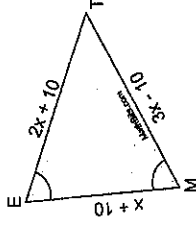


3. The vertex angle of an isosceles triangle measures 20 degrees more than twice the measure of one of its base angles. How many degrees are there in a base angle of this triangle?

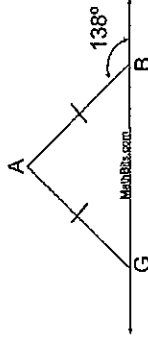
4. $\triangle DEG$ and $\triangle EGF$ are isosceles.
 $m\angle EDG = 64^\circ$
 Find $m\angle GEF$.



5. $m\angle MET = m\angle EMT$
 $ET = 2x + 10$ and $EM = x + 10$
 $MT = 3x - 10$
 Find MT .



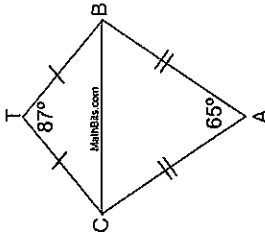
7. $\triangle GAB$ is isosceles. The exterior angle at point B measures 138° . Find $m\angle A$.



6. In $\triangle DEG$, $\overline{DE} \cong \overline{EG}$. An altitude is drawn from point E to \overline{DG} and intersects the side at H . Which of the following conclusions is NOT always true?

- 1) $\angle DEH \cong \angle HEG$
- 2) $\angle DHE \cong \angle GHE$
- 3) $\overline{DH} \cong \overline{HG}$
- 4) $\overline{DH} \cong \overline{EH}$

8. $\triangle CAB$ and $\triangle CTB$ are isosceles.
 $m\angle CTB = 87^\circ$ and $m\angle CAB = 65^\circ$.
 Find $m\angle ACT$.

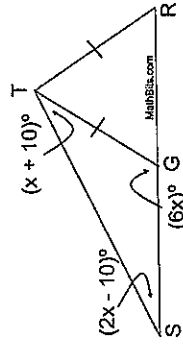


9. $\triangle ABC$ is isosceles with $AB = BC$. Which of the following statements is always true?

- 1) $m\angle C < m\angle B$
- 2) $m\angle B = m\angle A$
- 3) $m\angle A > m\angle B$

10. $TG = TR$; $m\angle S = 2x - 10$;
 $m\angle STG = x + 10$; $m\angle SGT = 6x$

- a) Find $m\angle S$.
- b) Find $m\angle TGR$.
- Let $GT = 3y + 6$ and $TR = 5y - 8$.
- c) Find TR .
- d) Find SR .



- ANSWERS
- 1. 10
 - 2. 96
 - 3. 40
 - 4. 32
 - 5. 50
 - 6. (4)
 - 7. 96
 - 8. 104
 - 9. (4)
 - 10. a) 30
 - b) 60
 - c) 27
 - d) 54