Geometry

Unit 1-3

Compositions and Constructions

of

Rigid Motions

Unit 3 Lesson 1:
Composition of Transformations

When two or more transformations are combined
to form a new transformation,
the result is called a
composition of transformations,
or a
sequence of transformations.

$$T_{1,5} \circ r_{y=x}$$

Do Second ⤴   ⤴ Do First

BEWARE This process must be done from right to left (←)!!

Composition of transformations is not commutative.

Example 1: $r_{y\text{-axis}} \circ T_{<3,-5>}$ (4, 3)

"A translation of (x+3, y-5) followed by a reflection in the y-axis."

(4, 3) ⟶ (   ) ⟶ (   )

Do not go back to pre-image! Refect new image!!

Example 2:
What is the image that results from this composition of transformations? $r_{x\text{-axis}} \circ R_{O,90}$ (-3, 0)

Example 3:

Find the coordinates of the image of (2, 4) under the transformation $r_{y\text{-axis}} \circ T_{2,-5}$

Example 4:
What is the image that results from this composition of transformations? $R_{O,180} \circ R_{O,90}(-2, 3)$

Example 5:
What is the image that results from this composition of transformations? $R_{O,90} \circ R_{O,90}(4, -1)$

Summary:

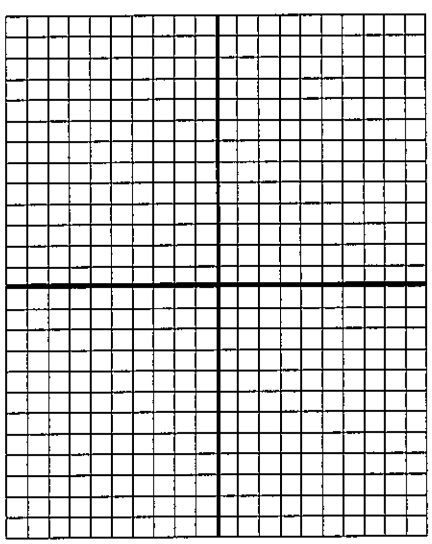
Example 6:
What is the image that results from this composition of transformations? $T_{\langle -3, 5\rangle} \circ T_{\langle 5, -1\rangle}(9, 0)$

Example 7:
What is the image that results from this composition of transformations? $T_{\langle 2, 7\rangle} \circ T_{\langle -1, 3\rangle}(3, -6)$

Summary:

Example 8:

The coordinates of ΔFUN are F(-5, 1), U(-1, 1), and N(-1, 7).



a. On a coordinate plane draw and label ΔFUN.

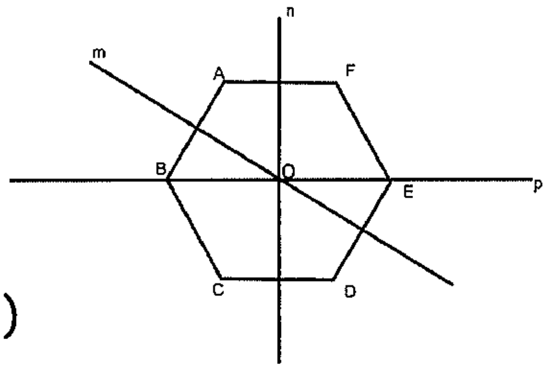
b. Draw and label ΔF'U'N', the image of ΔFUN after $r_{x\text{-axis}}$

c. Draw and label ΔF''U''N'', the image of ΔF'U'N' after a translation $\langle -1, 2\rangle$

d. Draw and label ΔF'''U'''N''', the image of ΔF''U''N'' after R_{90°.

e. Write the composition.

Example 9:



1. $r_m \circ r_n (A)$

2. $R_{O,120} \circ r_p (C)$

3. $r_n \circ r_p \overline{FE}$

Unit 3 Lesson 1 HW

COMPOSITION OF TRANSFORMATIONS

1 The point $(3,-2)$ is rotated 90° about the origin and then dilated by a scale factor of 4. What are the coordinates of the resulting image?

1) $(-12,8)$
2) $(12,-8)$
3) $(8,12)$
4) $(-8,-12)$

2 What is the image of point $A(4,2)$ after the composition of transformations defined by $R_{90^\circ} \circ r_{y=x}$?

1) $(-4,2)$
2) $(4,-2)$
3) $(-4,-2)$
4) $(2,-4)$

3 What is the image of point $(1,1)$ under $r_{x-axis} \circ R_{0,90^\circ}$?

1) $(1,1)$
2) $(1,-1)$
3) $(-1,1)$
4) $(-1,-1)$

4 What are the coordinates of point A', the image of point $A(-4,1)$ after the composite transformation $R_{90^\circ} \circ r_{y=x}$ where the origin is the center of rotation?

1) $(-1,-4)$
2) $(-4,-1)$
3) $(1,4)$
4) $(4,1)$

5 The coordinates of $\triangle JRB$ are $J(1,-2)$, $R(-3,6)$, and $B(4,5)$. What are the coordinates of the vertices of its image after the transformation $T_{2,-1} \circ r_{y-axis}$?

1) $(3,1), (-1,-7), (6,-6)$
2) $(3,-3), (-1,5), (6,4)$
3) $(1,-3), (5,5), (-2,4)$
4) $(-1,-2), (3,6), (-4,5)$

6 If the coordinates of point P are $(2,-3)$, then $(R_{90} \circ R_{180})(P)$ is

1) $(-2,3)$
2) $(-2,-3)$
3) $(3,-2)$
4) $(-3,-2)$

7 Find the coordinates of $r_{y-axis} \circ r_{y=x}(A)$ if the coordinates of A are $(6,1)$.

8 Find the coordinates of the image of $(2,4)$ under the transformation $r_{y-axis} \circ T_{3,-5}$.

9 What is the image that results from this composition of transformations?

$r_{x\text{-axis}} \circ R_{0,90^\circ}(-3,0)$

10 Find the coordinates of point $M(-1,3)$ under the composite $r_{y-axis} \circ R_{90^\circ}$.

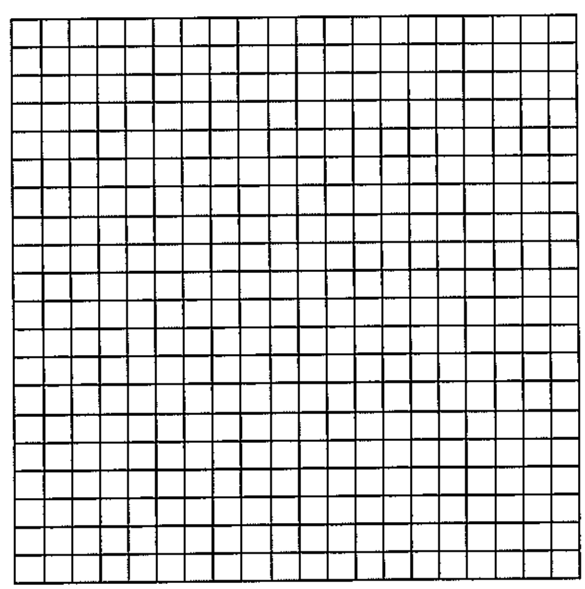
11 If the coordinates of A are $(2,-3)$, what are the coordinates of A', the image of A after $R_{90^\circ} \circ r_{y-axis}(A)$?

12 If the coordinates of B are $(1,-5)$, what are the coordinates of B', the image of B after $R_{90^\circ} \circ r_{x\text{-axis}} B$?

13 Find the image of point $A(3,-2)$ under the composition of translations $T_{2,1} \circ T_{-6,-4}$.

14 Write a single translation that is equivalent to $T_{3,-1}$ followed by $T_{-5,5}$.

15. On the accompanying grid, graph and label $\triangle ABC$ with vertices $A(3,1)$, $B(0,4)$, and $C(-5,3)$. On the same grid, graph and label $\triangle A''B''C''$. the image of $\triangle ABC$ after the transformation $r_{x-axis} \circ r_{y=x}$.



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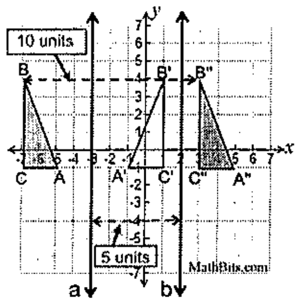
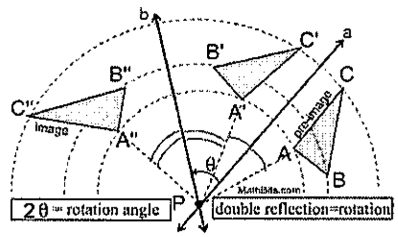
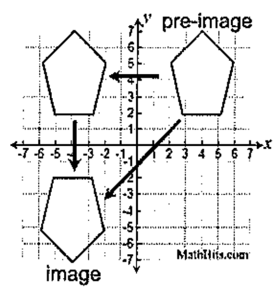
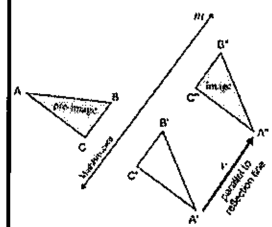
Unit 3 Lesson 2: Special Compositions

- In certain cases, a combination of transformations may be **renamed** by a single transformation.



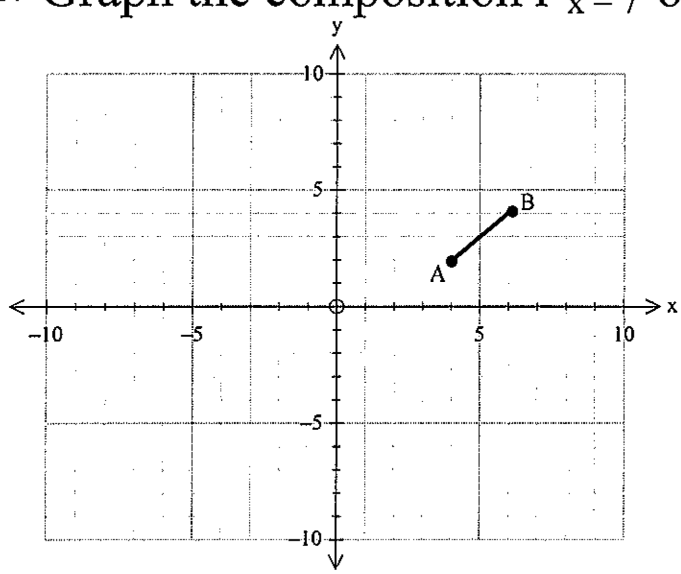






- A composition of reflections **over two parallel lines** is equivalent to a translation.

Example 1: Graph the composition $r_{x=7} \circ r_{x=2} (\overline{AB})$



What single transformation is the composition equivalent to?

A composition of line reflections in 2 parallel lines is equivalent to a TRANSLATION. → The translation will be DOUBLE THE DISTANC[E] between the parallel lines in the SAME direction as the composition.

Example 2:

Name the single transformation that would produce the same final image as the given composition:

(a) $r_{y=-5} \circ r_{y=-2}$

(b) $r_{y=-2} \circ r_{y=-5}$

(c) $r_{x=3} \circ r_{x=10}$

(d) $r_{x=10} \circ r_{x=3}$

Example 3:

Name the single transformation that would produce the same final image as the given composition:

(a) $r_{y=-1} \circ r_{y=-7}$

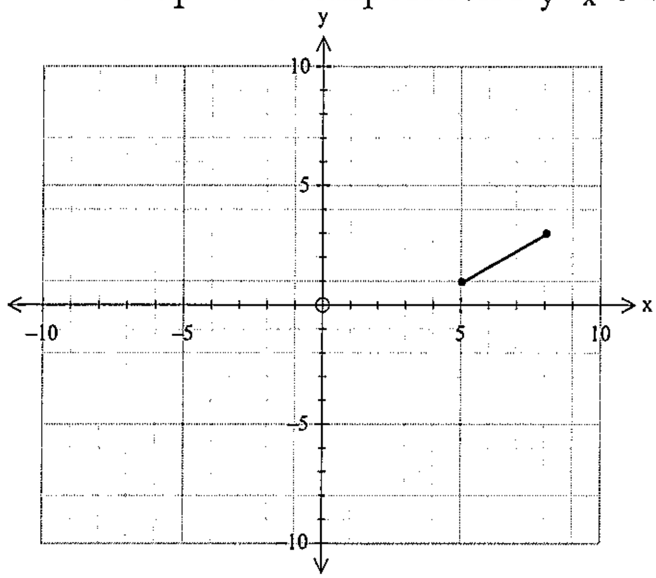
(b) $r_{y=-2} \circ r_{y=4}$

(c) $r_{x=-2} \circ r_{x=0}$

(d) $r_{x=6} \circ r_{x=-3}$

- The composition of reflections **over two intersecting lines** is equivalent to a rotation.

Example 4: Graph the composition $r_{y=x} \circ r_{x\text{-axis}} (\overline{AB})$

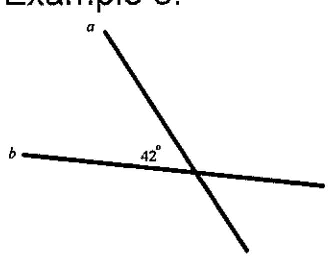


What single transformation is the composition equivalent to?

A composition of line reflections in intersecting lines is equivalent to a ROTATION.

The ROTATION will be DOUBLE THE ANGLE FORMED BY THE INTERSECTING LINES in the SAME direction as the composition.

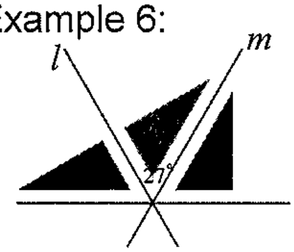
Example 5:



$r_b \circ r_a =$

$r_a \circ r_b =$

Example 6:

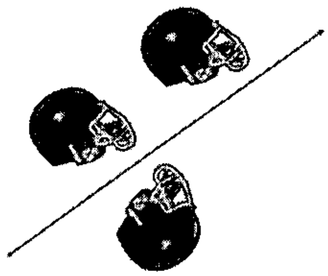


Write the composition of line reflections.

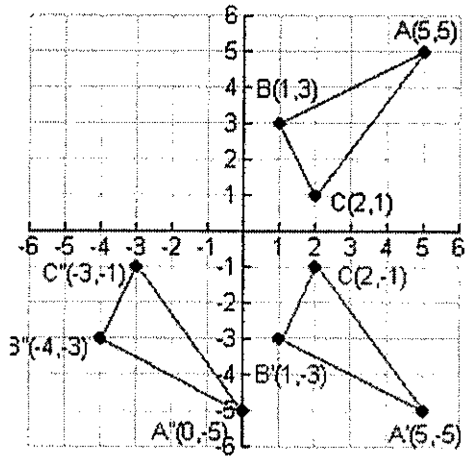
Write the single transformation that is equivalent to the composition.

Glide Reflections

A glide reflection is the composition of a line reflection and a translation.
The translation MUST be parallel to the line of reflection.
Glide reflections are commutative.



Is triangle $A''B''C''$ a glide reflection of triangle ABC?



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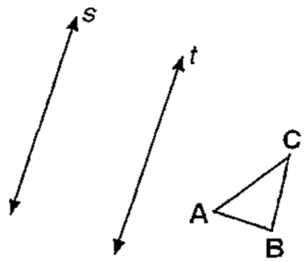
Special Compositions of Transformations

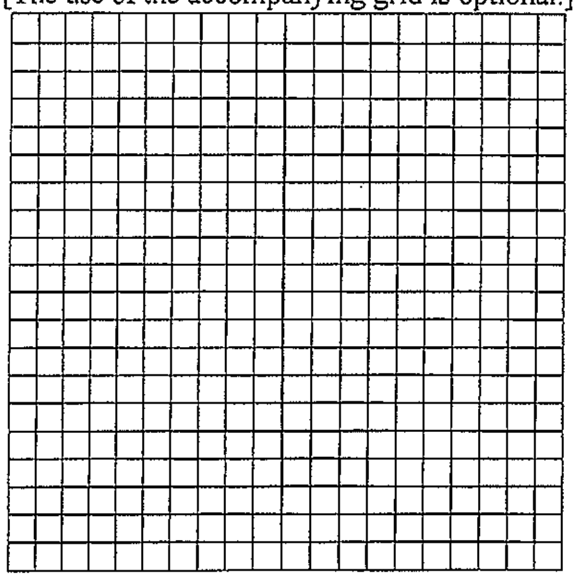
1) In the accompanying diagram, line s is parallel to line *t*.



Which is equivalent to the composition of line reflections $r_s \circ r_t$ (ΔABC)?

A) a glide-reflection B) a line reflection
C) a rotation D) a translation

2) Given point A(-2,3). State the coordinates of the image of A under the composition $T_{-3,-4} \circ r_{x-axis}$. [Show all work.]
[The use of the accompanying grid is optional.]



3) The composition of line reflections $r_{y=x} \circ r_{x=5}$ represents what type of isometry?

A) line reflection B) glide reflection
C) rotation D) translation

4) Which transformation is equivalent to the composite line reflections $r_{y-axis} \circ r_{y=x}(\overline{AB})$?

A) a translation B) a rotation
C) a glide reflection D) a dilation

5) Which of the following is equivalent to $T_{3,5} \circ T_{4,-4}$?

A) $T_{12,-20}$ B) $T_{1,-9}$ C) $T_{-1,9}$ D) $T_{7,1}$

6) Which of the following is equivalent to $T_{-2,5} \circ T_{2,3}$?

A) $T_{0,8}$ B) $T_{4,-2}$ C) $T_{-4,2}$ D) $T_{-4,15}$

7) If the coordinates of point P are (-5,9), then $(R_{30^\circ} \circ R_{45^\circ})(P)$ is equivalent to

A) $(R_{90^\circ} \circ R_{90^\circ})(P)$ B) $(R_{20^\circ} \circ R_{25^\circ})(P)$
C) $(R_{60^\circ} \circ R_{15^\circ})(P)$ D) $(R_{-20^\circ} \circ R_{75^\circ})(P)$

8) For any point (x,y), which transformation is equivalent to $R_{45^\circ} \circ R_{-135^\circ}$?

A) R_{-90° B) $r_{y=x}$ C) R_{90° D) r_{x-axis}

9) What is the image of P(-4,6) under the composite $r_{x=2} \circ r_{y-axis}$?

A) (-8,6) B) (0,6) C) (4,-2) D) (6,0)

10) The coordinates of point A are (3,-1). What are the coordinates of A′ under the transformation $(T_{2,5} \circ r_{x-axis})(A)$?

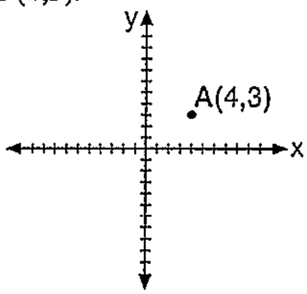
A) (5,6) B) (-5,-4) C) (5,4) D) (-1,4)

11) What are the coordinates of point A′, the image of point A(-4,1) after the composite transformation $R_{90^\circ} \circ r_{y=x}$ where the origin is the center of rotation?

A) (-4,-1) B) (1,4) C) (4,1) D) (-1,-4)

9

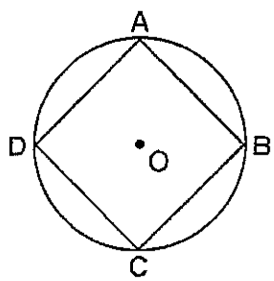
12) In the accompanying diagram, point A has coordinates (4,3).



What are the coordinates of A′, $r_{x=2} \circ r_{x=6}$ (A)?

A) (-4,3) B) (4,11) C) (12,3) D) (4,-5)

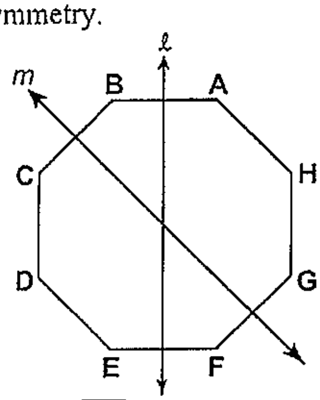
13) Square ABCD is inscribed in a circle with the center at O.



What is $(R_{-180^{\circ}} \circ R_{90^{\circ}})(B)$?

A) A B) B C) D D) C

14) In the accompanying figure, lines ℓ and m are lines of symmetry.



What is $r_m \circ r_\ell(\overline{BC})$?

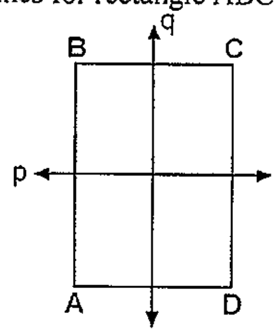
A) $\overline{GF}$ B) $\overline{DE}$ C) $\overline{BC}$ D) $\overline{HA}$

15) Lines ℓ and m intersect at point P. If the angle formed by the two lines is 50°, what is the angle of rotation equivalent to $r_\ell \circ r_m$?

A) 25° B) 100° C) 150° D) 50°

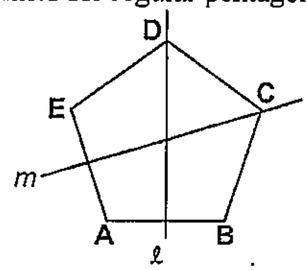
16) Write $(R_{-30^{\circ}} \circ R_{40^{\circ}} \circ R_{60^{\circ}})(A)$ as an equivalent single rotation of A.

17) In the accompanying diagram, p and q are symmetry lines for rectangle ABCD.



Find $r_p \circ r_q \circ r_p$ (A).

18) In the accompanying figure, ℓ and m are symmetry lines for regular pentagon ABCDE.

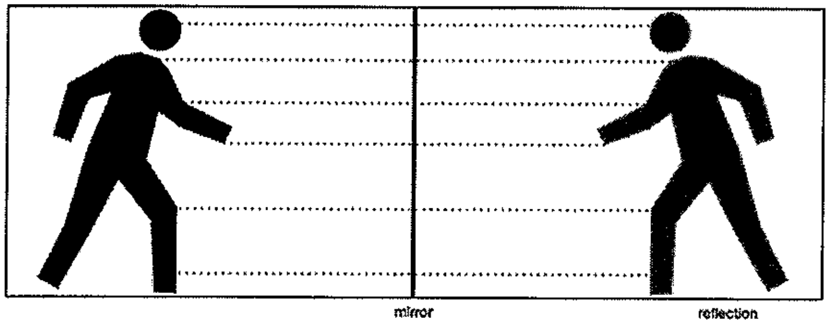


Find $r_\ell \circ r_m$ (A).

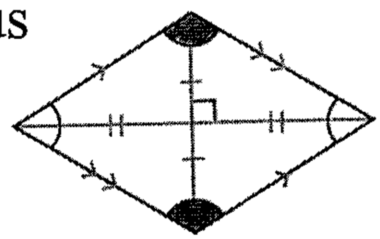
19) (a) On graph paper, draw the graph of the parabola $y = (x+3)^2 - 2$ for all values of x in the interval $-6 \leq x \leq 0$.
(b) On the same set of axes, draw the image of the graph drawn in part (a) after a translation that maps (x,y) to $(x+3, y+2)$.
(c) On the same set of axes, draw the image of the graph drawn in part (b) after a reflection in the x-axis.

Unit 3 Lesson 3

Constructing Line Reflections

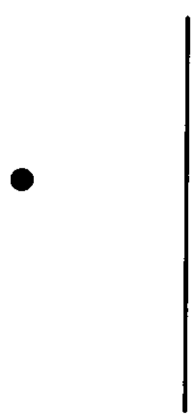


The Rhombus



Example 1

Reflecting a point across a line



How to construct a REFLECTION

a) Given $\overline{AB}$ and a line of reflection.

b) Placing your compass pointer at A, extend the compass to reach past the line of reflection so that when you make an arc you create two intersections (in our case at D and E).

c) Leaving your compass with the same measurement that you just used to create D and E, place your pointer at D and create an arc on the opposite side of A.

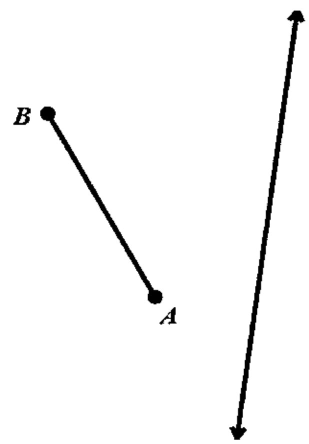
d) Do the same thing but from E. Place your pointers at E and with that same measurement create an arc that intersects the arc that you just made from D. The intersection of these two arcs is A'

e) Repeat the steps b-d but from B. Placing your compass at B, extend your compass so that it reaches beyond the line of reflection. This will create two intersections (H and G in our case).

f) Now from H create an arc of the congruent length on the opposite side of B.

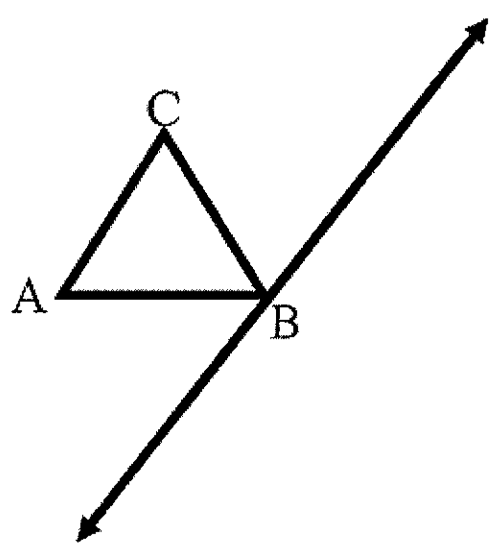
Example 2

Reflecting a segment across a line



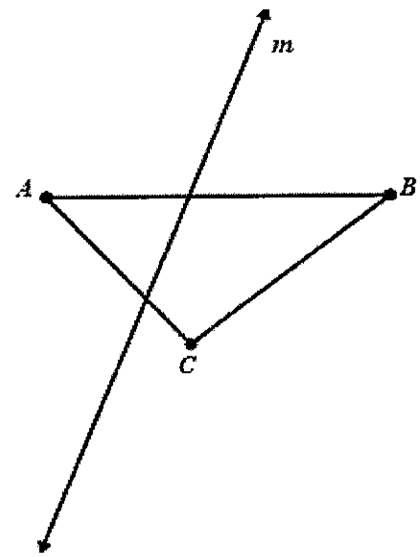
Example 3

Using a straightedge and a compass construct the following line reflection.



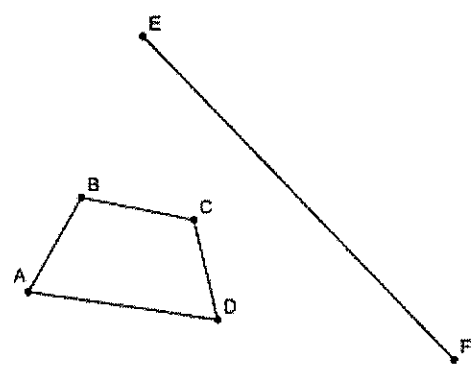
Example 4

Using a straightedge and a compass construct the following line reflection.



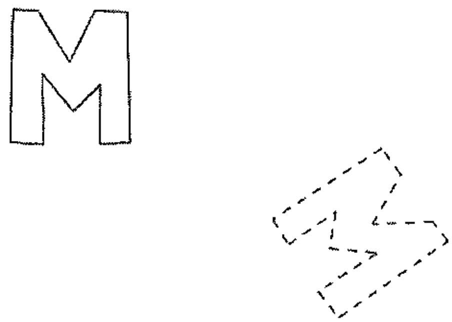
Example 5

Using a straightedge and a compass construct the following line reflection.



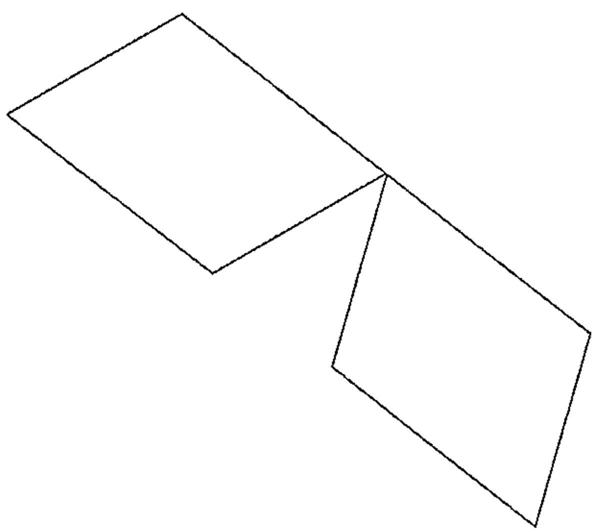
Example 6

Using a straightedge and a compass construct the line of reflection.



Example 7

Using a straightedge and a compass construct the line of reflection.



Name: ______________________________

1) Reflecting (5,1) in the y-axis yields an image of

A) (-5,1) C) (5,1)
B) (5,-1) D) (-5,-1)

2) The point (-3,-2) is reflected in the origin. The coordinates of its image are

A) (-3,2) C) (-2,-3)
B) (3,2) D) (2,3)

3) If the point (2,-5) is reflected in the line $y = x$, then the image is

A) (-5,-2) C) (-5,2)
B) (-2,5) D) (5,-2)

4) When point A(-2,5) is reflected in the line $x = 1$, the image is

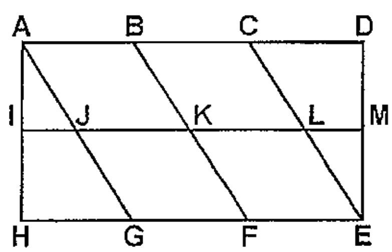
A) (0,5) C) (-2,-3)
B) (4,5) D) (5,2)

5) The transformation $T_{-2,3}$ maps the point (7,2) onto the point whose coordinates are

A) (5,-1) C) (5,5)
B) (9,5) D) (-14,6)

6) A translation maps the point (5,-2) to a point (0,-2). What is the image of the point (0,-2) under the same translation?

7) In the accompanying diagram, K is the image of A after a translation.



Under the same translation, which point is the image of J?

8) $\triangle ABC$ has vertices A(-5,4), B(-2,3) and C(6,-1). Find the coordinates of the images of the vertices of $\triangle ABC$ under the given glide reflection.

$T_{5,0} \circ r_{x\text{-axis}}$

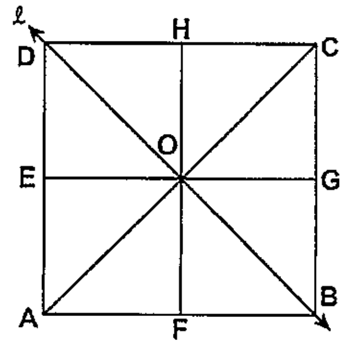
9) Isometries that are opposite isometries are a

A) single line reflection, only
B) glide reflection and a rotation
C) single line reflection and a rotation
D) single line reflection and a glide reflection

10) Which transformation does *not* preserve orientation?

A) $r_{y=x}$ C) $T_{5,-3}$
B) D_3 D) $R_{0,90°}$

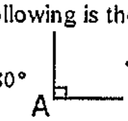
11) In the accompanying diagram of square ABCD, F is the midpoint of $\overline{AB}$, G is the midpoint of $\overline{BC}$, H is the midpoint of $\overline{CD}$, and E is the midpoint of $\overline{DA}$.

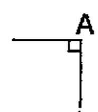


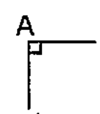
(a) Find the image of $\triangle EOA$ after it is reflected in line ℓ.

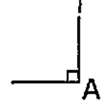
(b) Is this isometry direct or opposite? [*Explain your answer.*]

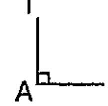
12) Which of the following is the image of

$R_{A,90°} \circ R_{A,-180°}$  ?

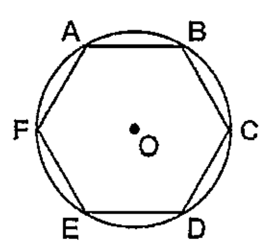
A) 

C) 

B) 

D) 

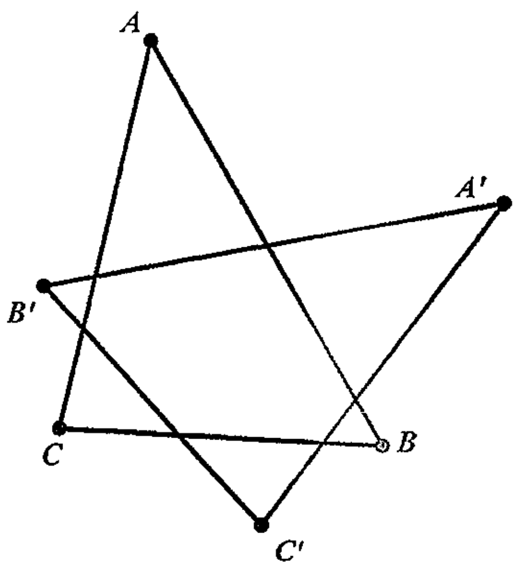
13) In the accompanying diagram, regular hexagon ABCDEF is inscribed in circle O.



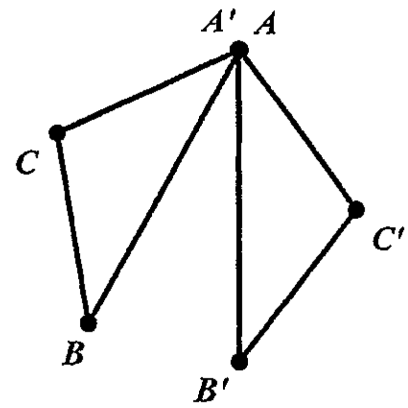
With O as the center of rotation, find $R_{O,-120°} \circ R_{O,240°}$ (A).

15

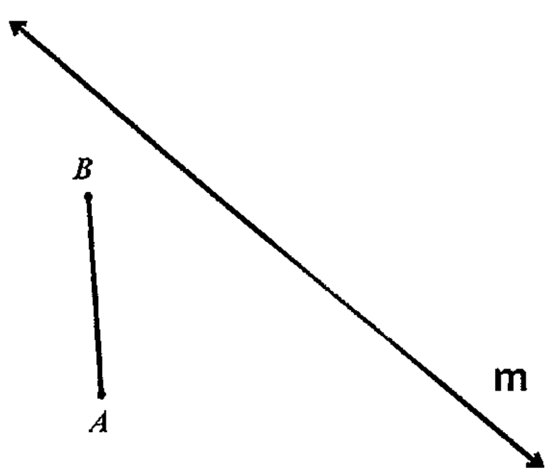
14. Construct the line of reflection



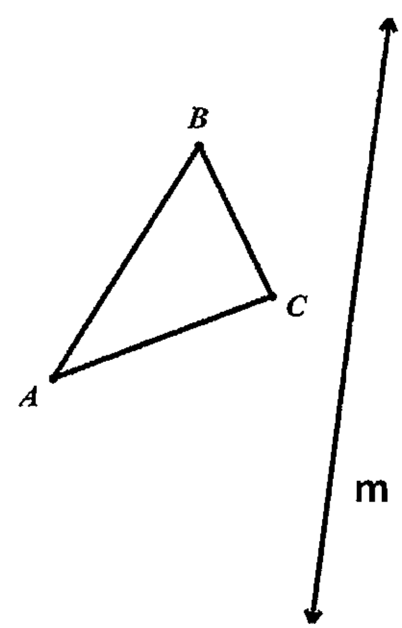
15. Construct the line of reflection



16. Construct the line reflection below.

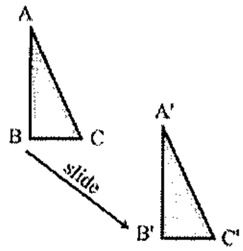


17. Construct the line reflection below.

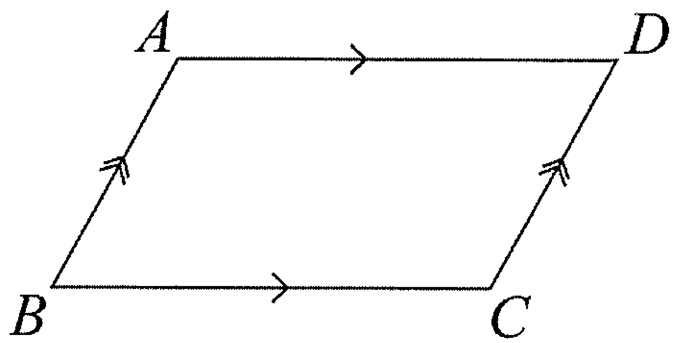


Unit 3 Lesson 4

Constructing Translations



The Parallelogram

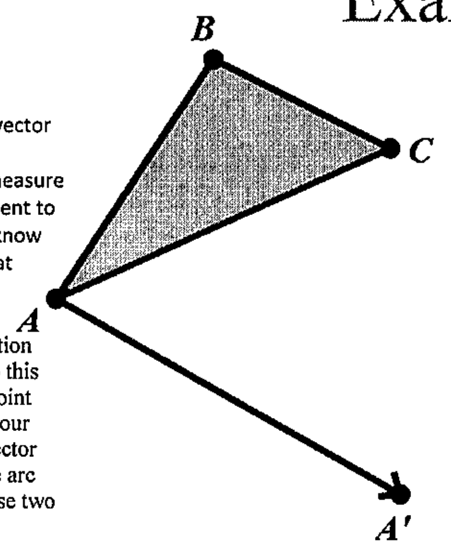


Example 1:

Translating a Point Along a Vector

How to construct a TRANSLATION

Example 2:



a) Given a ΔABC and vector $\overline{AA'}$.

b) In a translation all points move by vector AA' (the same direction and the same distance) so we use our compass to measure the distance AA'. Use that measurement to make an arc from B and from C. We know that B and C must move that far in that general direction.

c) Next we want to find the exact location of B' on the arc we just created. To do this measure the distance from the initial point of the vector to B. Move the point of your compass to the terminal point of the vector and create an arc that will intersect the arc drawn from B. The intersection of these two arcs forms B'.

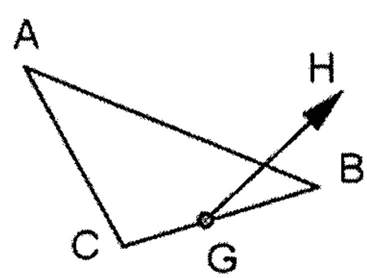
d) Repeat to find the location of C'

e) Connect A'B', B'C' and C'A' to complete the triangle.

Example 3:

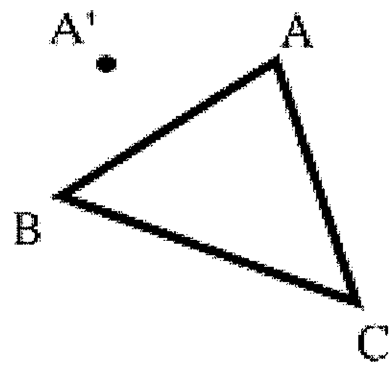
Use a compass and a straightedge to construct the following translation.

Apply $T_{\overrightarrow{GH}}$ to translate Δ ABC



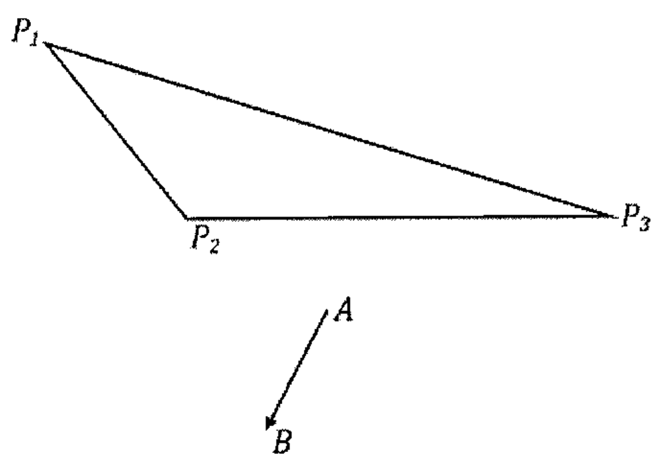
Example 4: **Use a compass and a straightedge to construct the following translation.**

$T_{\overrightarrow{AA'}}(\Delta ABC)$



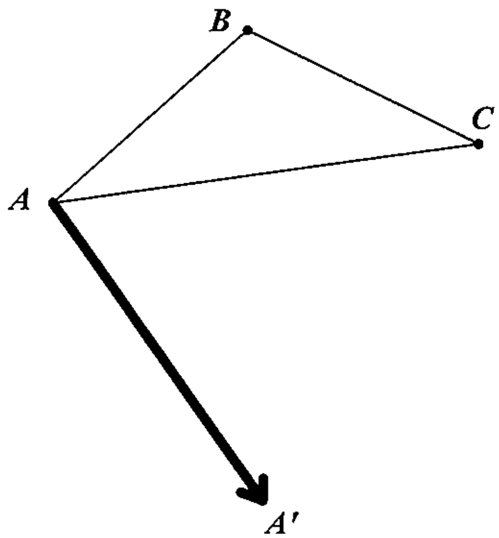
Example 5: **Use a compass and a straightedge to construct the following translation.**

Apply $T_{\overrightarrow{AB}}$ to $\Delta\ P_1P_2P_3$

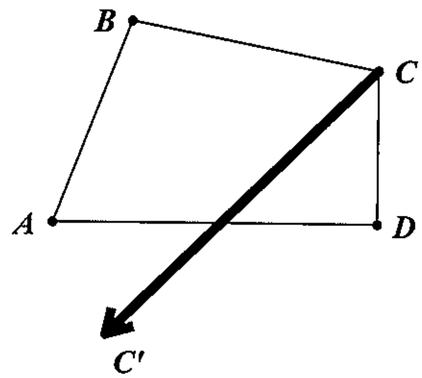


1. Use a compass and a straightedge to construct the following translations.

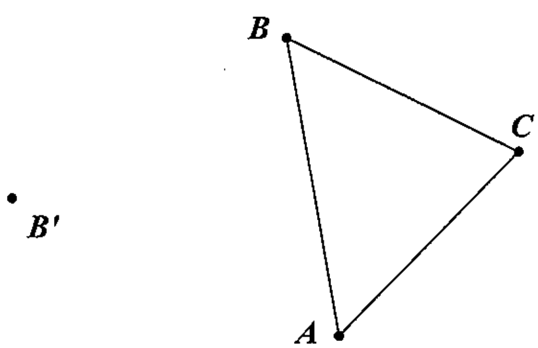
$T_{\overline{AA'}}(\Delta ABC)$



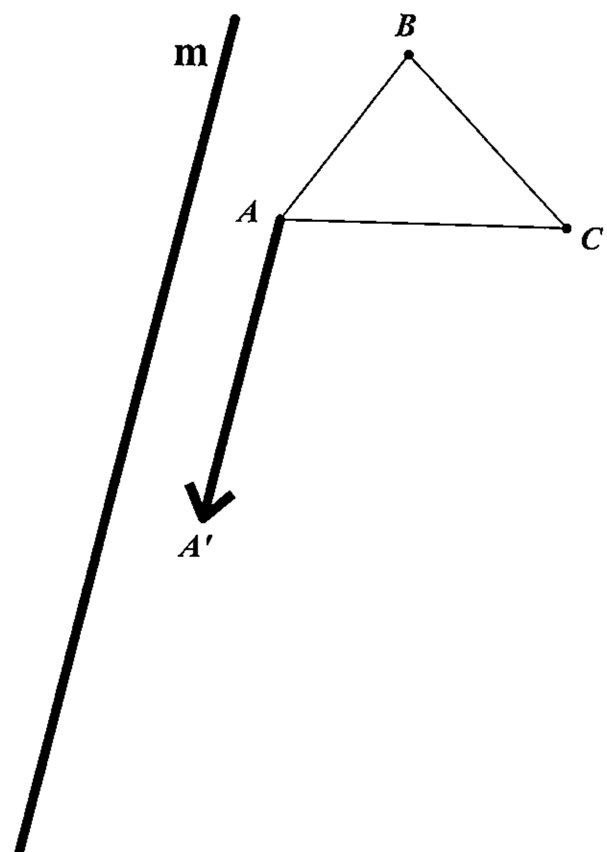
$T_{\overline{CC'}}(ABCD)$



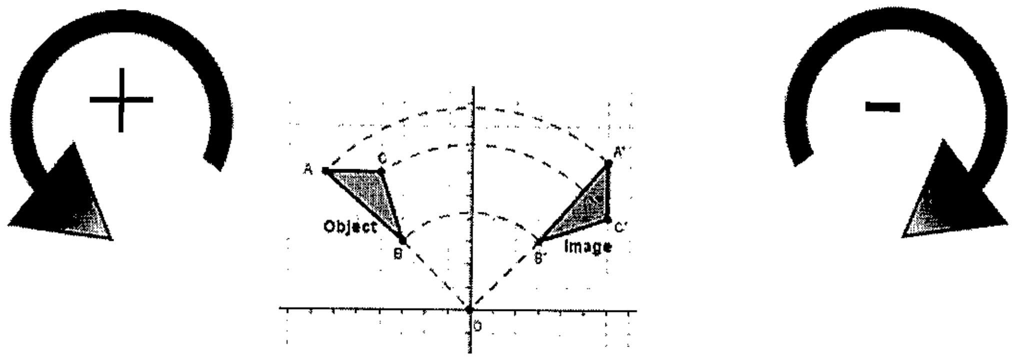
$T_{\overline{BB'}}(\Delta ABC)$



$T_{\overline{AA'}}(\Delta ABC)$ **AND THEN** $r_m(\Delta A'B'C')$



Unit 3 Lesson 5
Constructing Rotations

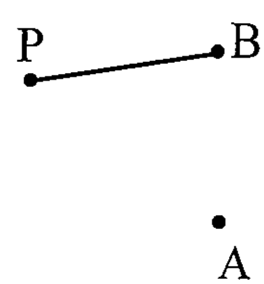


$60^\circ, 30^\circ, 90^\circ, 45^\circ, 180^\circ$

How could you construct $75^\circ, 150^\circ, 210^\circ, 270^\circ$?

Example 1:

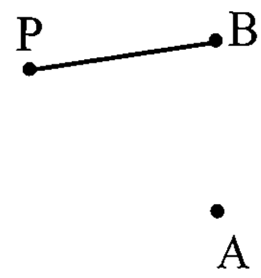
- Construct the following rotation, label image P'B':
  $R_{A, 60^\circ}(\overline{PB})$



Example 2:

- Construct the following rotation, label image P'B':

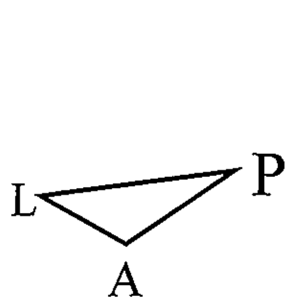
  $R_{A, 30^\circ}(\overline{PB})$



Example 3:

- Construct the following rotation, label image L':

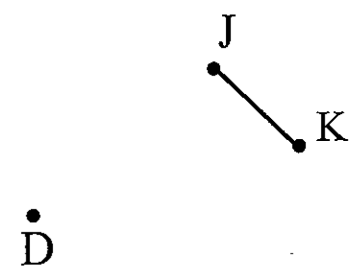
  $R_{K, 180^\circ}(\Delta LAP)$



Example 4:

- Construct the following rotation, label image J':

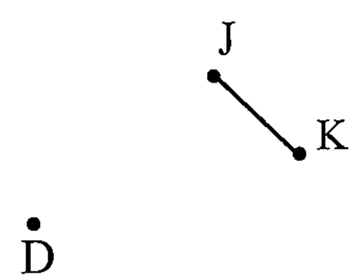
  $R_{D, 90^\circ}(\overline{JK})$



Example 5:

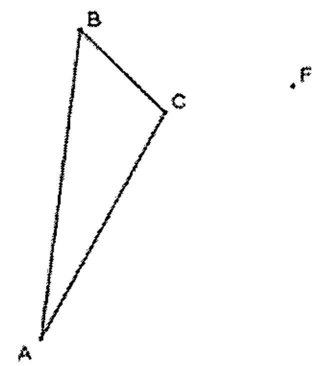
- Construct the following rotation, label image J':

  $R_{D, 45^\circ}(\overline{JK})$



Example 6:

Rotate the triangle ABC -60^{o} around point F using a compass and a straightedge only.



Steps for Constructing a Rotation:

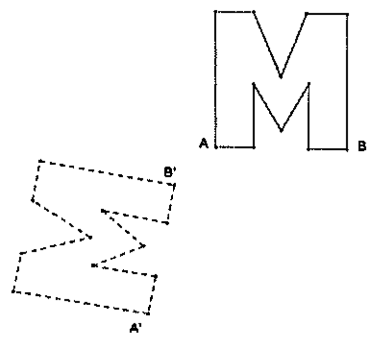
1. Put your compass point on the center of rotation and make an arc going through the point that you are reflecting.

2. Determine the construction necessary for the angle that you are given.

- 60 Equilateral Triangle
- 30 Equilateral Triangle then bisect the angle
- 90 Perpendicular from a point on the line (center of rotation)
- 45 Perpendicular from a point on the line then bisect the angle
- 180 Extend line through center. (EASIEST ONE)

Example 7:

Finding the Center of Rotation



Steps for finding the center of rotation:

a. Draw a segment connecting points A and A'.

b. Using a compass and straightedge, find the perpendicular bisector of this segment.

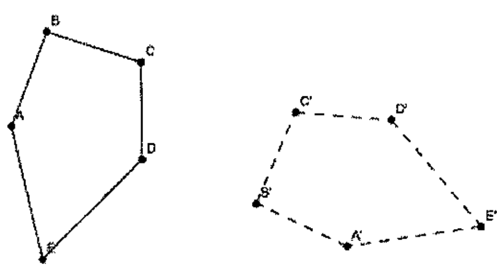
c. Draw a segment connecting points B and B'.

d. Find the perpendicular bisector of this segment.

e. The point of intersection of the two perpendicular bisectors is the center of rotation. Label this point P.

Example 8:

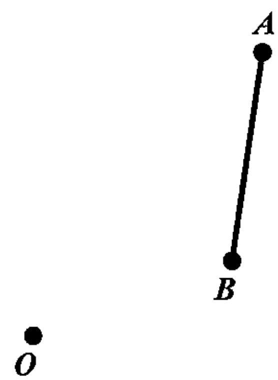
Using your compass and straightedge, find the center of rotation.



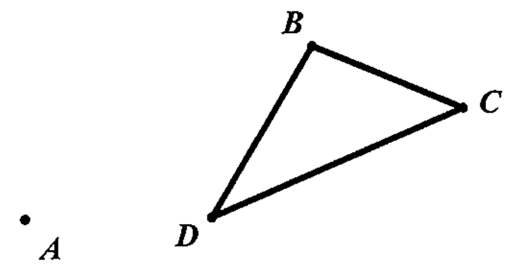
Date ____________________________________

Unit 5 Lesson 5 HW : Constructing Transformations (ALL) Homework

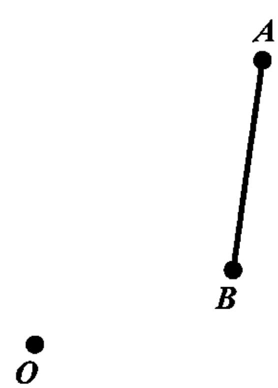
1. Construct: $R_{O,60^o}(\overline{AB})$



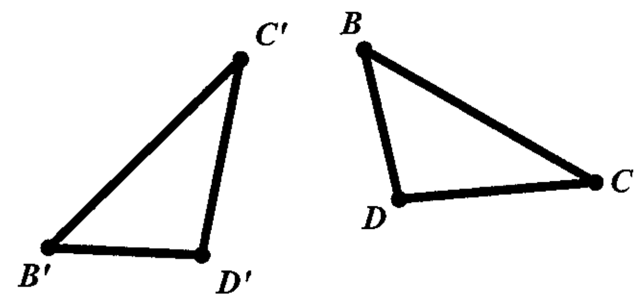
2. Construct: $R_{A,180^o}(\Delta DBC)$



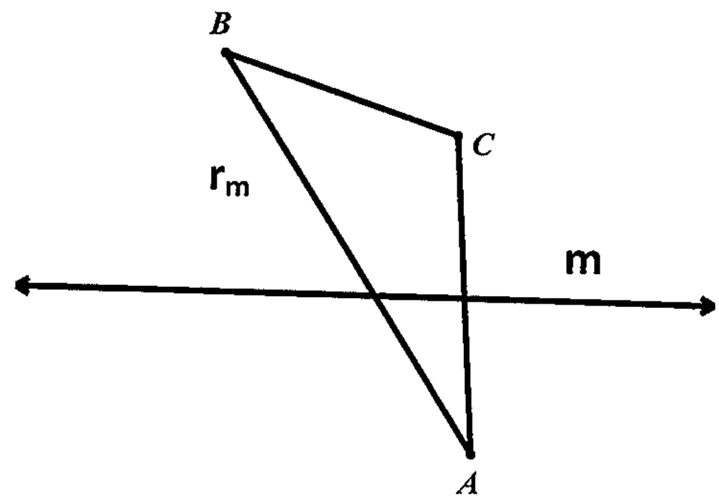
3. Construct: $R_{O,90^o}(\overline{AB})$



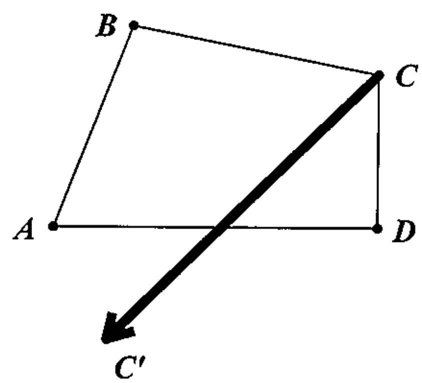
4. Use constructions to find the center of rotation.



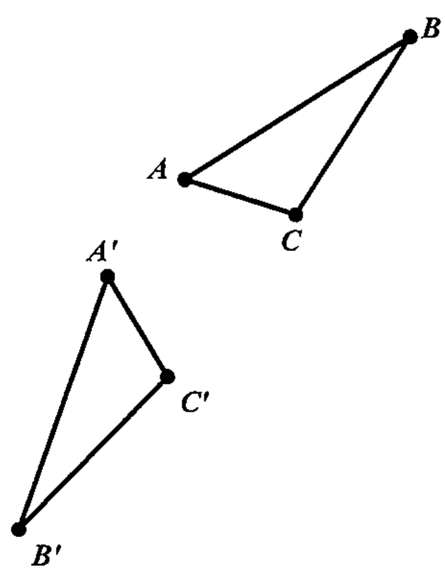
5. Construct the following line reflection:



6. Construct: $T_{\overline{CC'}}(ABCD)$



7. Use constructions to find the refection line.

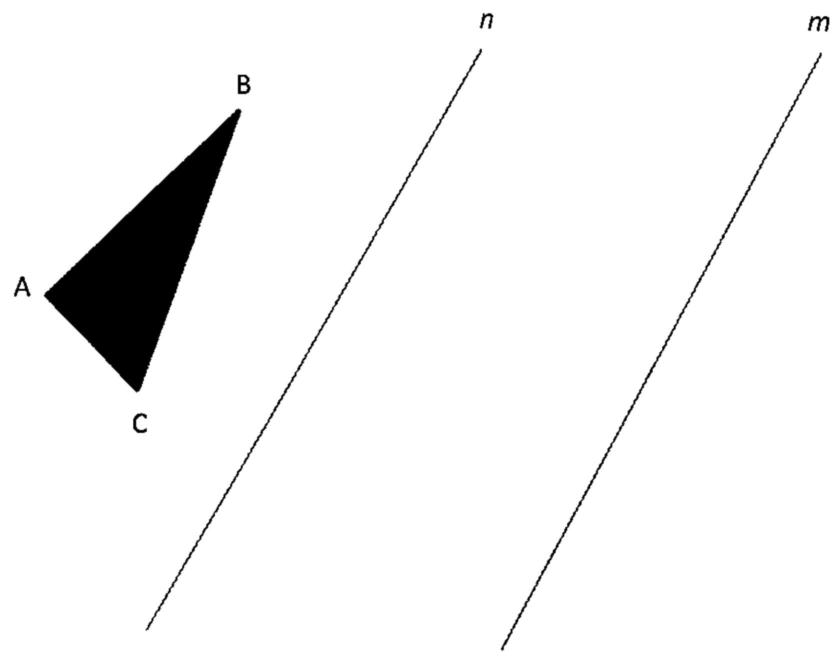


Date ____________ Geometry

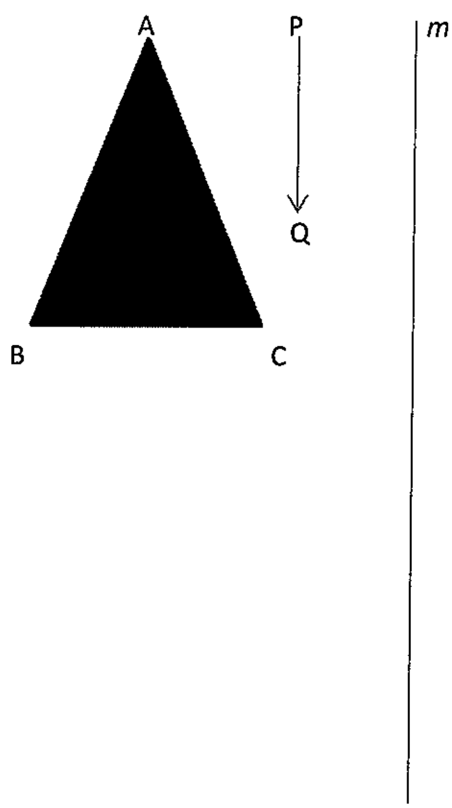
Unit 3 Lesson 6: Constructing Compositions of Rigid Motions

1. Perform the following composition:

   $r_m \circ r_n$ (ΔABC)

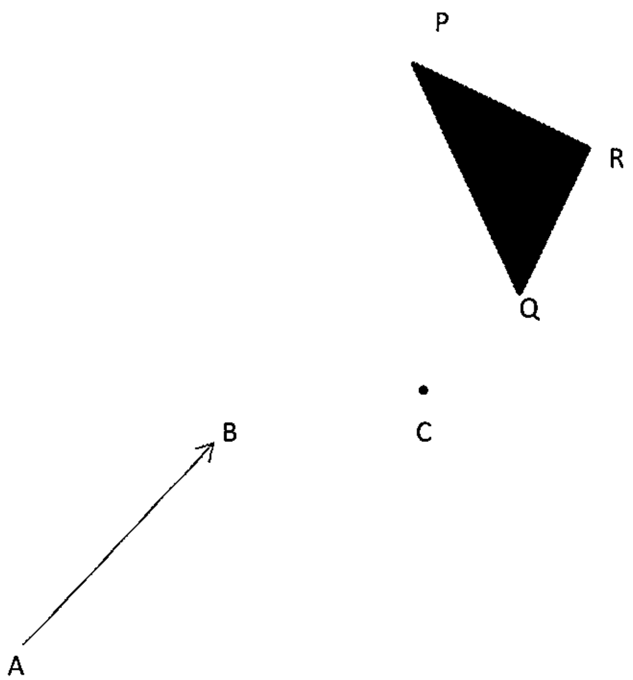


2. Construct the following composition: $r_m \circ T_{\overrightarrow{PQ}}$ (ΔABC)

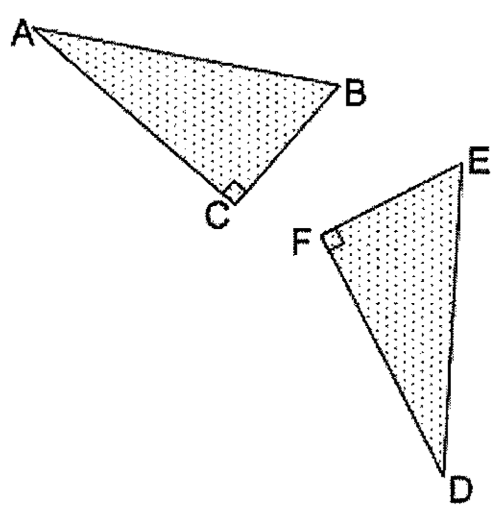


What is this transformation called?

3. Construct the following composition: $T_{\overrightarrow{AB}} \circ R_{C,\,60^\circ}(\Delta PQR)$



4. Describe the sequence of rigid motions that maps ΔABC onto ΔDEF.
Not a Construction!



Date ________ Geometry

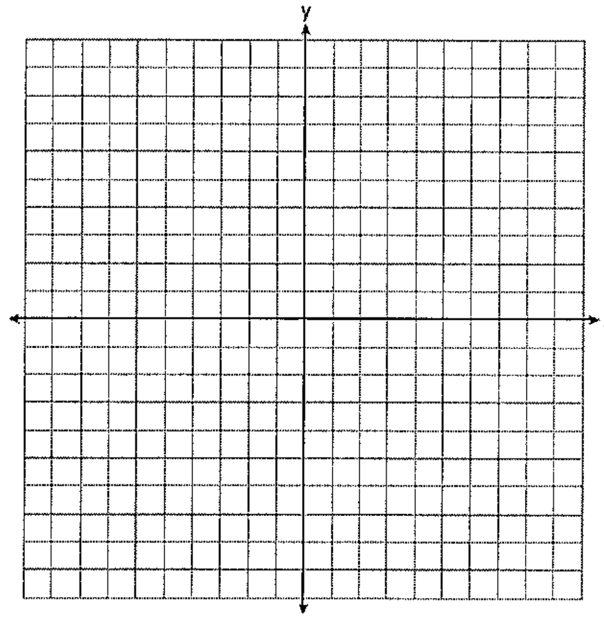
Unit 1-3 REVIEW: Compositions and Constructions of RIGID MOTIONS

1. What are the coordinates of point A', the image of point $A(-4, 1)$ after the composite transformation $R_{90°} \circ r_{y=x}$ where the origin is the center of rotation?

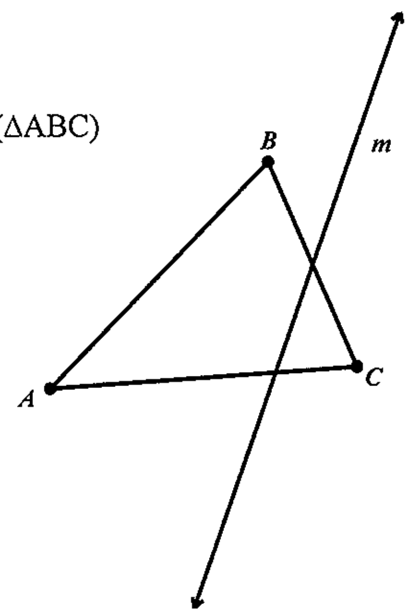
2. The coordinates of $\triangle JRB$ are $J(1,-2)$, $R(-3,6)$, and $B(4,5)$. What are the coordinates of the vertices of its image after the transformation $T_{2,-1} \circ r_{y\text{-axis}}$?

3. Write a single translation that is equivalent to $T_{3,-1}$ followed by $T_{-5,5}$.

4. Triangle ABC has coordinates $A(-3,-7)$, $B(-3,-3)$, and $C(0,-3)$.
   - *a* On the graph below, graph and label $\triangle ABC$.
   - *b* Graph and state the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$ after a point reflection in the origin.
   - *c* Graph and state the coordinates of $\triangle A''B''C''$, the image of $\triangle A'B'C'$ reflected in the line $y = 2$.
   - *d* Graph and state the coordinates of $\triangle A'''B'''C'''$, the image of $\triangle A''B''C''$ after translation $T_{(-8,2)}$.



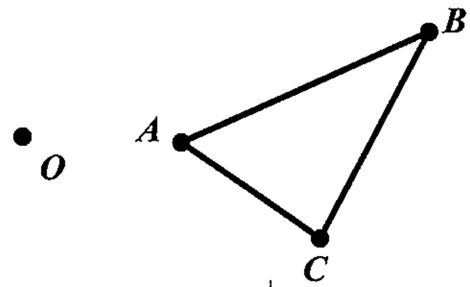
5. Construct the following $r_m(\triangle ABC)$



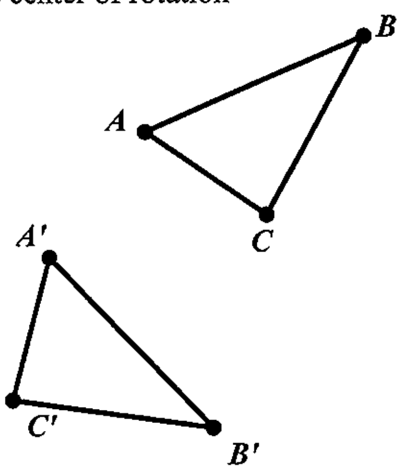
6. A double reflection over x = 11 followed by x = 7, translates all points right 8 units. T or F
7. A double reflection over x = 1 followed by x = -4, translates all points left 10 units. T or F
8. $r_{x=1} \circ r_{x=-2} = T_{<0, 6>}$ T or F
9. A double reflection over y = -3 followed by y = -1, translates all points up 4 units. T or F
10. A composition of a translation and a rotation is an opposite isometry T or F
11. A double line reflection in y = 6 followed by y = -1, translates all points left 14 T or F
12. $r_{x=2} \circ r_{x=6} = T_{<-4, 0>}$ T or F

13. If you wanted to rotate a shape by 38° by double reflecting it over two intersecting lines, the angle between the two intersecting lines would need to be ________.

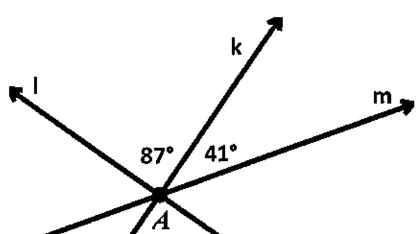
14. Construct the following: $R_{O,120^\circ}(\Delta ABC)$



15. Find the center of rotation

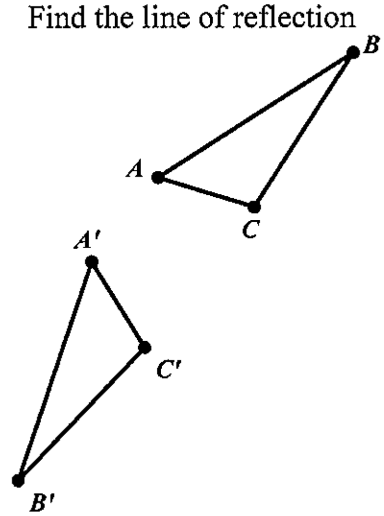


16.



In the diagram above, $r_m \circ r_k$ of any point, P is equivalent to what single transformation?

17. Find the line of reflection



18. Use a compass and straight edge to construct the following translation.