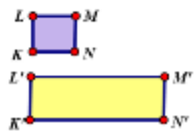
Unit 2 Transformations and RIGID MOTIONS

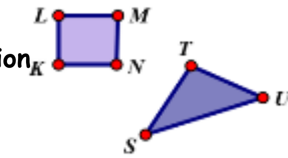
Summary Sheet

1. **<u>Pre-image</u>**: the shape that you start with.
2. **<u>Image</u>**: The new shape after the mapping (transformation) takes place.
3. A **<u>TRANSFORMATION</u>** is like a one-to –one function from algebra. A ONE TO ONE FUNCTION is when there is exactly the same number of elements in the domain as there is in the range. A **<u>transformation</u>** in geometry is when you have the same number of points in the pre-image as in the image.

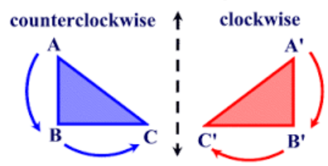
   1 Example of Transformation



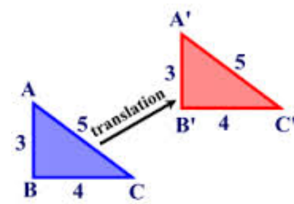
   1 NON-Example of a Transformation



4. **<u>An ISOMETRIC TRANSFORMATION</u>** or **<u>RIGID MOTION</u>** is a transformation that results in an image that is *CONGRUENT* to the pre-image.
   - PRESERVED: Distances, angle measures, parallelism, colinearity,
5. **<u>Orientation</u>** is the direction (clockwise/counterclockwise) in which the vertices are lettered.

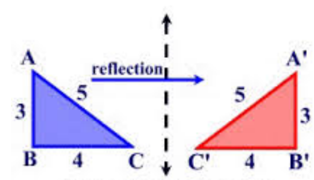


   - **<u>Direct Isometry</u>** is when orientation is preserved from pre-image to image.



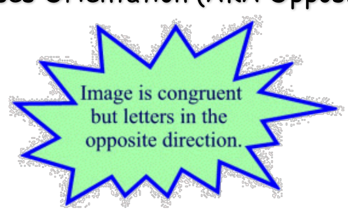
Direct Isometry

   - **<u>Opposite Isometry</u>** is when orientation is reversed from pre-image to image. LINE REFLECTIONS REVERSE ORIENTATION!



Opposite Isometry

6. **<u>TYPES OF ISOMETRIC TRANSFORMATIONS:</u>**
   - **a. <u>Line Reflections</u>**: Always Reverses Orientation (AKA Opposite Isometry)
     - $r_{x\text{-axis}}(x, y) \rightarrow (x, -y)$
     - $r_{y\text{-axis}}(x, y) \rightarrow (-x, y)$
     - $r_{y=x}(x, y) \rightarrow (y, x)$
     - $r_{y=-x}(x, y) \rightarrow (-y, -x)$



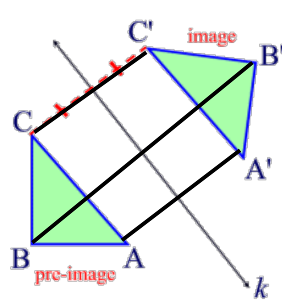
You can always count the number of spaces (perpendicular to the line of reflection) and then continue that number of spaces to your image point.

     - reflection in any other line, graph and count!

   Points on line of reflection are **INVARIANT**

   Pre-image to Image PATHS are: Parallel, Not Equal.

x = # is a VERTICAL LINE
Y = # is a HORIZONTAL LINE

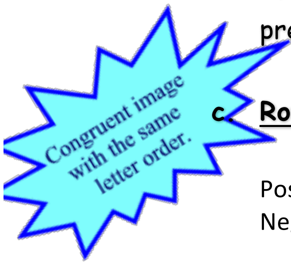


b. **Point Reflections**: Same as a rotation of 180°. Always PRESERVES Orientation. (AKA Direct Isometry)

- r_{origin} or r_o $(x, y) \rightarrow (-x, -y)$
- reflection in any other point, graph and count!

Point of reflection should be the midpoint between the pre-image and image.

You can always count the number of spaces (to the point of reflection) and then continue that number of spaces to your image point.



c. **Rotations:** Always PRESERVES orientation. (AKA Direct Isometry)

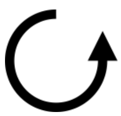
Positive rotations Counter Clockwise
Negative rotations Clockwise

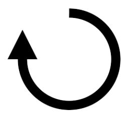
- $R_{O,90}\ (x, y) \rightarrow (-y, x)$
- $R_{O,180}\ (x, y) \rightarrow (-x, -y)$
- $R_{O,270}\ (x, y) \rightarrow (y, -x)$

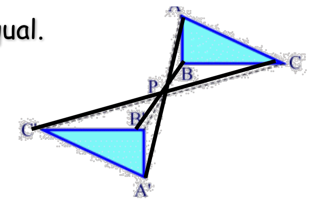
You can always rotate your paper and note the new coordinates. Then return your paper to the normal position and plot the image point.

Points on center of rotation are **INVARIANT**

Pre-image to Image PATHS are: NOT Parallel, Not Equal.


(+) Rotation COUNTER-CLOCKWISE


(-) Rotation CLOCKWISE



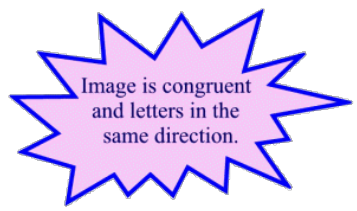
Equivalent Rotations: example R_{90} is equivalent to R_{-270}. The images will be in the same location!

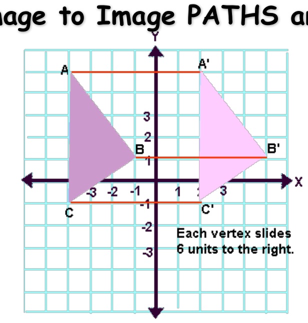
d. **Translations:** Always Direct Isometry.

- $T_{a,b}\ (x, y) \rightarrow (x + a, y + b)$
- < a, b > is vector notation for a translation.

NO Invariant Points

Pre-image to Image PATHS are: Parallel, Equal

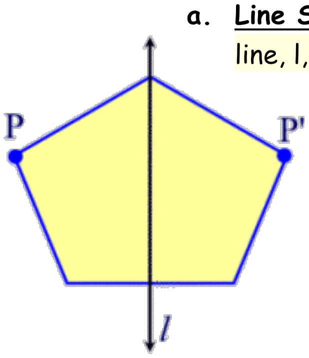




You can always count the translation.
x + # moves RIGHT
x - # moves left
Y + # moves UP
Y - # moves DOWN

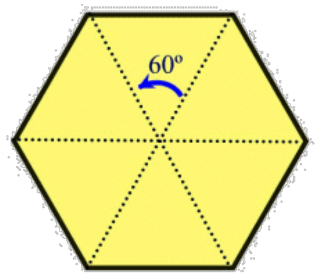
7. **SYMMETRY:**

a. **Line Symmetry (reflectional symmetry)** A set of points has **line symmetry** if and only if there is a line, l, such that the reflection through l of each point of the set is also a point of the set.



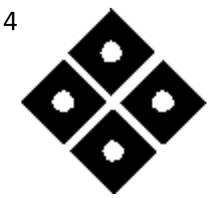
b. **<u>Rotational Symmetry</u>** An object has **rotational symmetry** if there is a center point around which the object is turned (rotated) a certain number of degrees and the object looks the same.

- **<u>Regular Polygons</u>** $\frac{360}{\#\ of\ sides} = degree\ of\ rotational\ symmetry$



- **<u>Order</u>** The number of positions in which the object looks unchanged.

Example of Order 4



c. **<u>Point Symmetry</u>**: exists when a figure is built around a single point called the center of the figure. For every point in the figure, there is another point found directly opposite it on the other side of the center.

A simple test to determine whether a figure has point symmetry is to turn it upside-down and see if it looks the same. A figure that has point symmetry is unchanged in appearance by a 180 degree rotation.

8. **<u>Quadrants</u>**