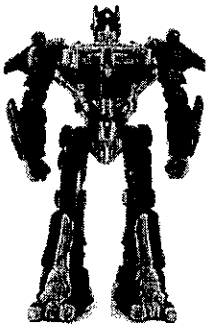

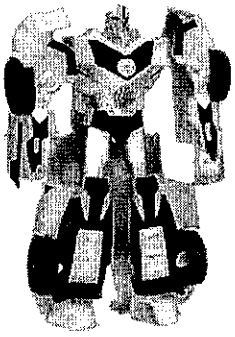

Geometry

Unit 1-2

Transformations and Rigid Motions

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Lesson 2: Line Reflections	pages 4-7	HW pages 8-9
Lesson 3: Rotations and Point Reflections	pages 10-14	HW pages 15-16
Lesson 4: Translations	pages 17-19	
Review	pages 20-22	

Unit 2 Lesson 1 :
Transformations ⟶ RIGID MOTIONS

Transformations take points in the plane as inputs and give other points as outputs. As such, transformations behave like "functions".

The name of transformation "function"

INPUT OUTPUT

$T_{a,b}(x,y) = (x+a, y+b)$

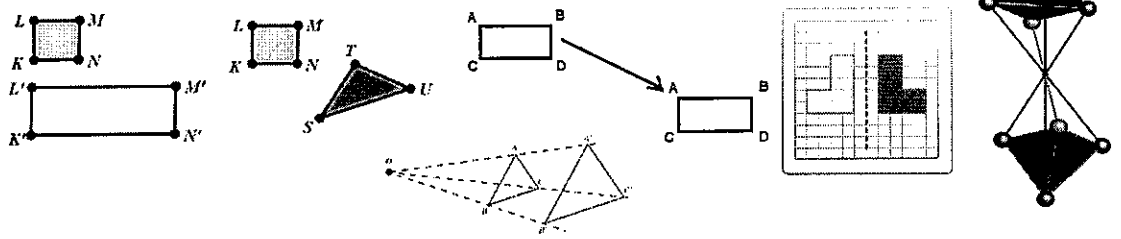
MathBits.com

Example 1:
Given coordinate rule G (x, y) ------ > (2x, y + 1), determine the image of B (5,2)?

Example 2:
Given coordinate rule T (x, y) ------ > (x + 3, y – 6), determine the pre-image of C' (3, 2)?

Example 3:
Given coordinate rule G (x, y) ------ > (2x, y + 1), determine the pre-image of D' (-4, 11)?

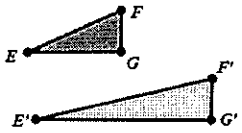
In a <u>transformation</u> the pre-image
and the image
will have the SAME NUMBER OF POINTS

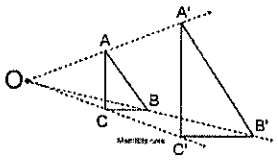


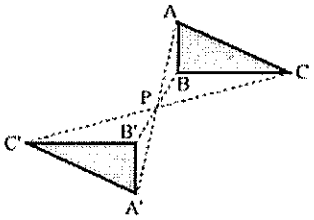
In a <u>RIGID</u> transformation
(AKA rigid motion, AKA Isometry)
the pre-image is congruent to the image

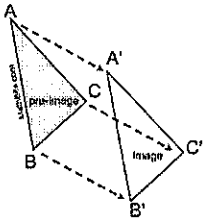
Types of Transformations

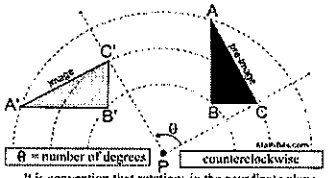
1. Translation
2. Line Reflection
3. Point Reflection
4. Rotation
5. Glide Reflection
6. Dilation
7. Stretch

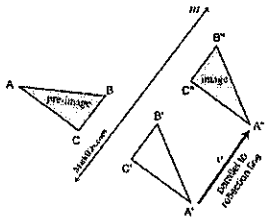


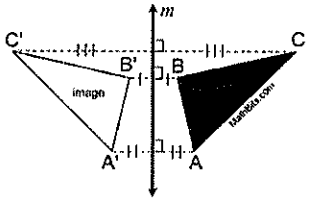










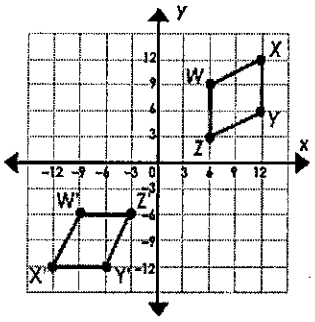


Properties of RIGID MOTIONS

Since a line reflection, point reflection/rotation and translation are isometric transformations (Rigid Motions) the following properties are preserved between the pre-image and its image:

- Distance (lengths of segments are the same)
- Angle measure (angles stay the same)
- Parallelism (things that were parallel are still parallel)
- Collinearity (points on a line, remain on the line)

*After a reflection, rotation and translation, the pre-image and image are identical.



Unit 2 Lesson 1 HW

1. Use the given coordinate rules to solve missing coordinates.

a) T (x,y) ------ > (x, y + 7) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (5,0)

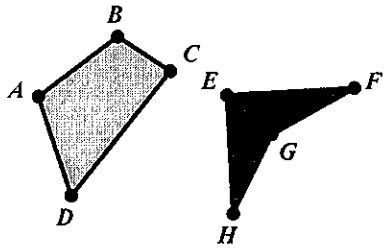
b) S (x,y) ------ > (-y, x) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (9,7)

c) F (x,y) ------ > (5x, 3y) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (-5,12)

2. Complete the Analogy. **Input** IS TO **Output** AS **Pre-Image** IS TO _____.

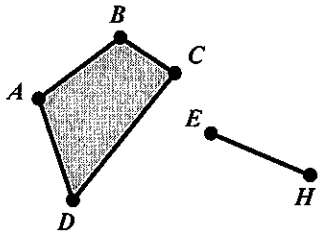
3. Given that the pre-image is Quadrilateral ABCD.

a)



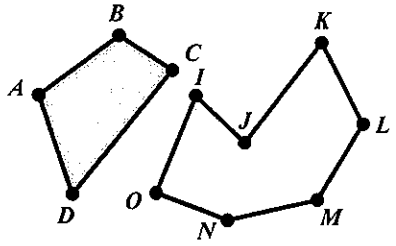
Transformation? YES or NO
Rigid Motion? YES OR NO

b)



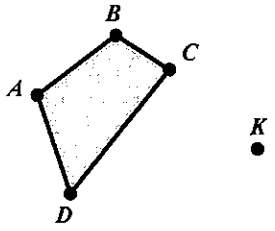
Transformation? YES or NO
Rigid Motion? YES OR NO

c)



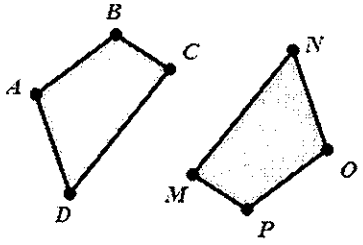
Transformation? YES or NO
Rigid Motion? YES OR NO

d)



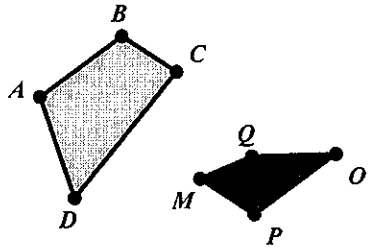
Transformation? YES or NO
Rigid Motion? YES OR NO

e)



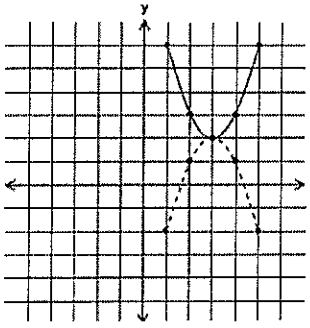
Transformation? YES or NO
Rigid Motion? YES OR NO

f)



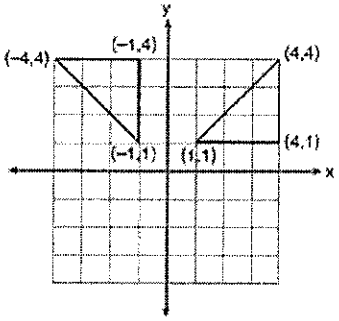
Transformation? YES or NO
Rigid Motion? YES OR NO

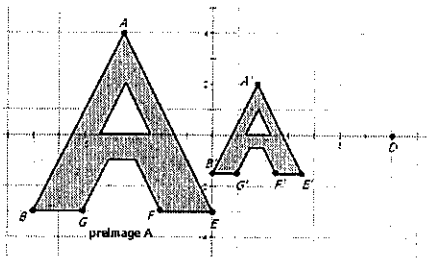
4. Name that transformation:



2

2





P q

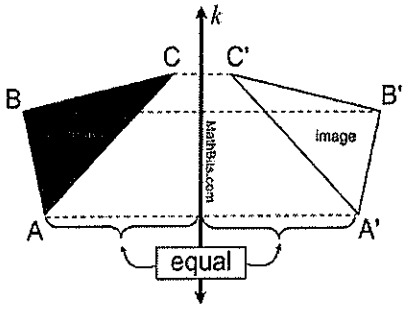
3

Unit 2 Lesson 2: Line Reflections

Properties preserved under a **line reflection** from the pre-image to the image.

1. **distance** (lengths of segments remain the same)
2. **angle measures** (remain the same)
3. **parallelism** (parallel lines remain parallel)
4. **collinearity** (points remain on the same lines)

The **orientation** (lettering around the outside of the figure), is not preserved. The order of the lettering in a reflection is reversed (from clockwise to counterclockwise or vice versa).

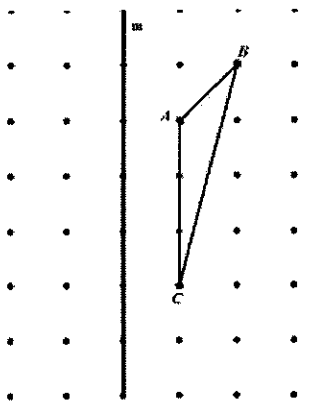


Relationship of Pre-image to Image	Line Reflection
Distance of the paths	Different (not congruent)
Relationship of the paths	parallel
Orientation	Different

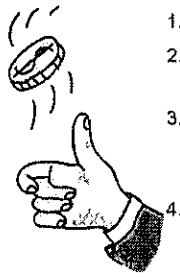
Orientation (lettering): The lettering of the points of the pre-image, in this diagram, is clockwise *A-B-C*, while the image is lettered counterclockwise *A'-B'-C'*. When lettering changes direction, in this manner, the transformation is referred to as a *non-direct* or *opposite isometry*.

Special Points	the point ON the line of reflection doesn't move

Example 1



How to do a Reflection



1. Plot the point(s).
2. Count from the point to the line of reflection.(Perpendicular)
3. Count the same number of spaces in the opposite direction.
4. Plot the image.

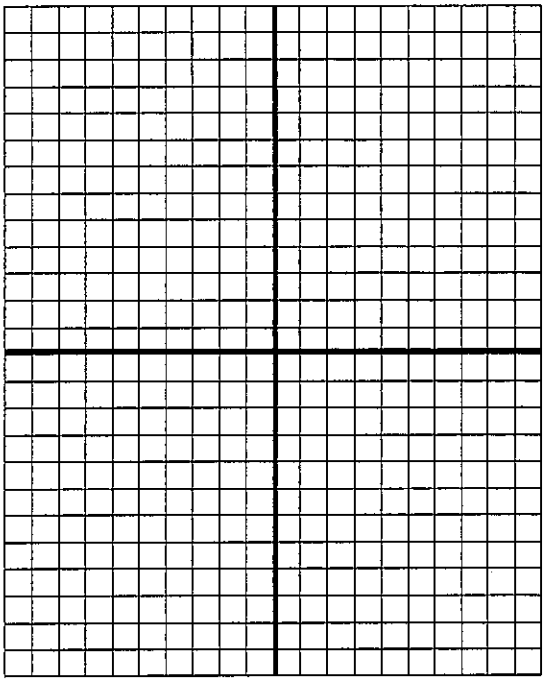
Common Reflections:

$r_{x-axis}(x,y) = (x,-y)$

$r_{y-axis}(x,y) = (-x,y)$

$r_{y=x}(x,y) = (y,x)$

$r_{y=-x}(x,y) = (-y,-x)$

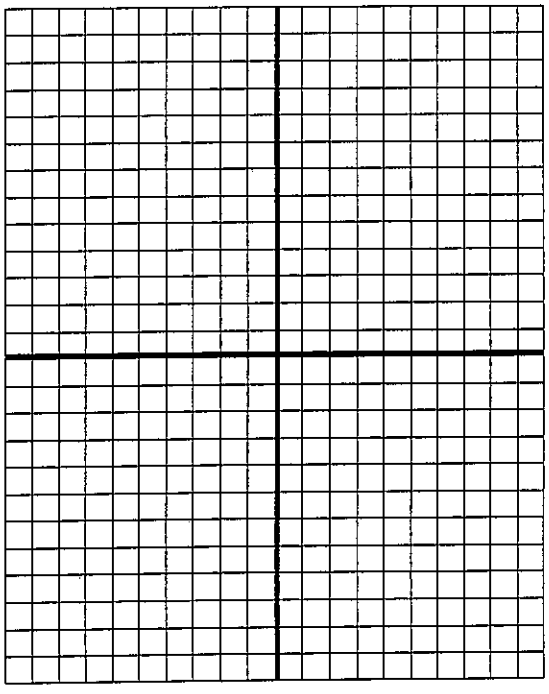


Example 2

Plot the point A (-3, 5)

- $r_{x\text{-axis}}(A)$
- $r_{y\text{-axis}}(A)$
- $r_{y = x}(A)$
- $r_{y = -x}(A)$

Example 3 Reflections over vertical and horizontal Lines

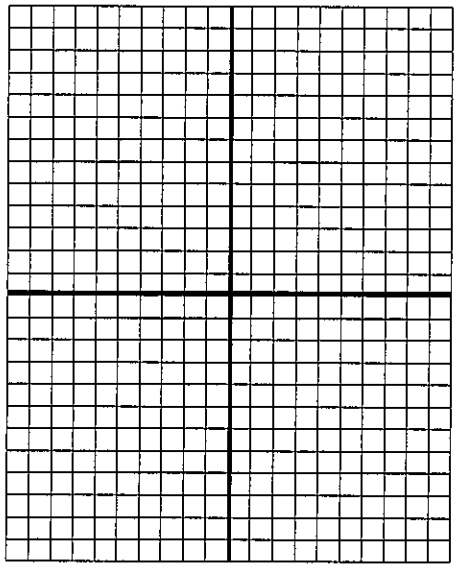


Plot point E (-3, -4)

- $r_{x = 3}(E)$
- $r_{y = -2}(E)$
- $r_{x = -3}(E)$

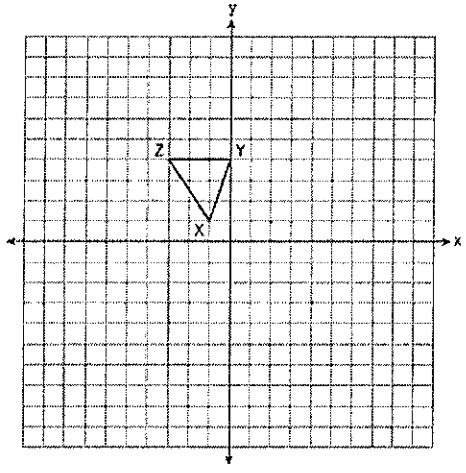
Example 4

Triangle ABC has vertices A(-1, 1), B(1, 3) and C(4, 1).
The image of ΔABC after the transformation $r_{y=x}$ is ΔA'B'C'.
State and label the coordinates of ΔA'B'C'.

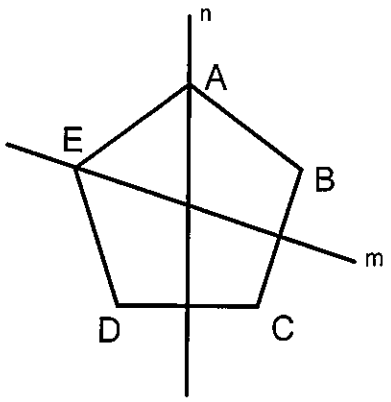


Example 5

Triangle XYZ shown in the diagram below is reflected over the line x = 2.
State and label the coordinates of ΔX'Y'Z', the image of ΔXYZ.



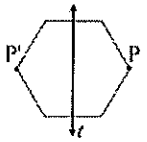
Example 6

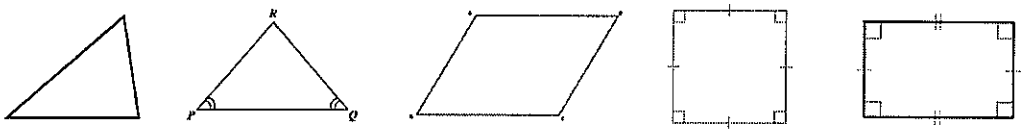


1. $r_m(C) = (\quad)$
2. $r_n(D) = (\quad)$
3. $r_n(E) = (\quad)$
4. $r_n(A) = (\quad)$
5. $r_n(\overline{AB}) = (\quad)$
6. $r_m(\overline{DE}) = (\quad)$

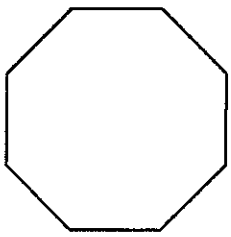
Line Symmetry

Definition: A set of points has line symmetry if and only if there is a line, *l*, such that the reflection through *l* of each point in the set is also a point in the set. (May also be referred to as *reflectional symmetry*.)



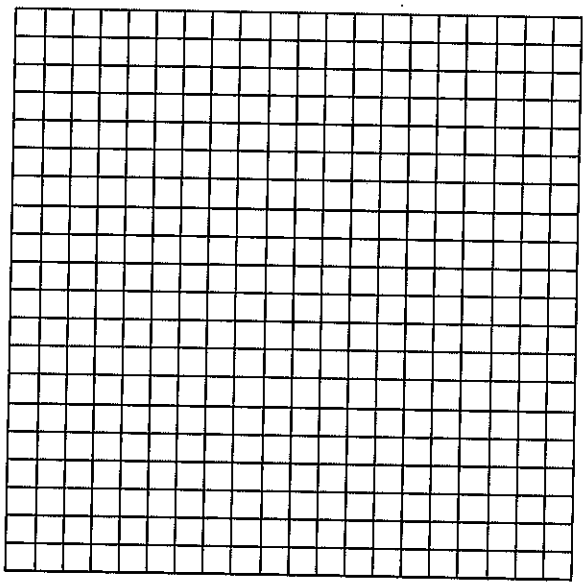


In regular polygons (where all sides are congruent and all angles are congruent). the number of lines of symmetry equals the number of sides.



Unit 2 Lesson 2 Line Reflections HW

1. Triangle SUN has coordinates $S(0,6)$, $U(3,5)$, and $N(3,0)$. On the accompanying grid, draw and label $\triangle SUN$. Then, graph and state the coordinates of $\triangle S'U'N'$, the image of $\triangle SUN$ after a reflection in the line y = -1.



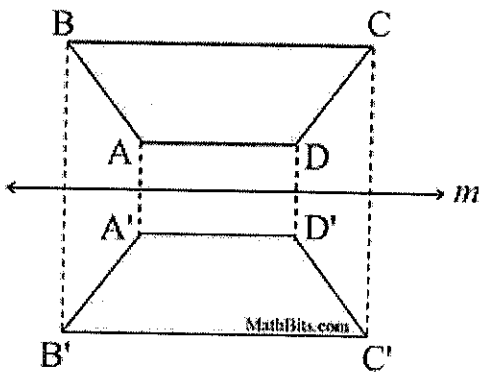
2. Point $(-2, 3)$ is reflected in the x-axis. In which quadrant does its image lie?
   1) I
   2) II
   3) III
   4) IV

3. If the point $(2, -5)$ is reflected in the line y = -x, then the image is
   1) $(5, -2)$
   2) $(-2, 5)$
   3) $(-5, 2)$
   4) $(-5, -2)$

4. What are the coordinates of point P, the image of point $(3, -4)$ after a reflection in the line $y = x$?
   1) $(3, 4)$
   2) $(-3, 4)$
   3) $(4, -3)$
   4) $(-4, 3)$

5. Reflecting $(5, 1)$ in the y-axis yields an image of
   1) $(5, -1)$
   2) $(-5, -1)$
   3) $(5, 1)$
   4) $(-5, 1)$

6. ABCD *is reflected* in line m as shown in the accompanying diagram. Circle the statement that is NOT true?



$\overline{DD'} \perp m$ $\quad$ $\overline{AB} \parallel \overline{A'B'}$

$\overline{BC} \cong \overline{B'C'}$ $\quad$ $\overline{BB'} \parallel \overline{CC'}$

7. During a reflection, $RR' = 18$ units. What is the distance from point R to the line of reflection?

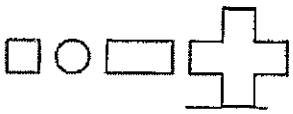
8. When working with reflections, which of the following statements is TRUE?
(1) The line of reflection is perpendicular to the segment connecting a pre-image point to its image.
(2) The line of reflection bisects the segment connecting a pre-image point to its image.
(3) The line of reflection intersects the segment connecting a pre-image point and its image at its midpoint.
(4) all of the above

9. Which figure always has exactly one line of symmetry

A) circle B) rectangle

C) trapezoid D) isosceles right triangle

10. Which kind of symmetry do all of these figures have?



A) neither vertical line nor horizontal line

B) vertical line, only

C) horizontal line, only

D) both vertical line and horizontal line

11. Which letter has vertical line symmetry?

A) P B) E C) T D) S

12. Which combination of letters has both vertical and horizontal line symmetry?

A) MOM B) ZOO

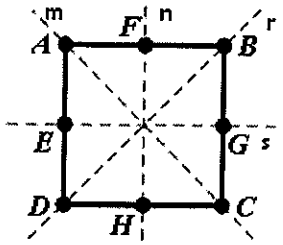
C) XOOX D) NOON

13. a) $r_n(A) =$ _____ b) $r_r(C) =$ _____

c) $r_s(D) =$ _____ d) $r_m($_____$) =$ B

e) $r_n(D) =$ _____ f) $r_m(E) =$ _____

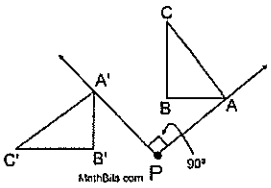
g) $r_m(G) =$ _____ h) $r_s($_____$) =$ H

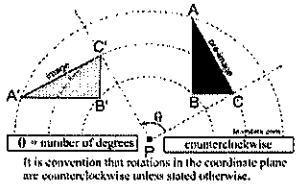


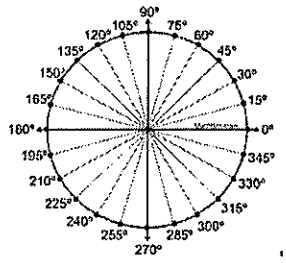
Unit 2 Lesson 3
Rotations and Point Reflections

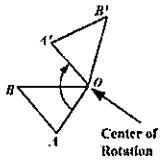
Properties preserved under a **rotation** from the pre-image to the image.
1. **distance** (lengths of segments remain the same)
2. **angle measures** (remain the same)
3. **parallelism** (parallel lines remain parallel)
4. **collinearity** (points remain on the same lines)
5. **orientation** (lettering order remains the same)

Relationship of Pre-image to Image	Rotation
Distance of the paths	Different (not congruent)
Relationship of the paths	not parallel
Orientation	Same
Special Points	the point at the center of rotation doesn't move









Common Graph Rotations:
(center at the origin)

$$R_{90°}(x, y) = (-y, x)$$

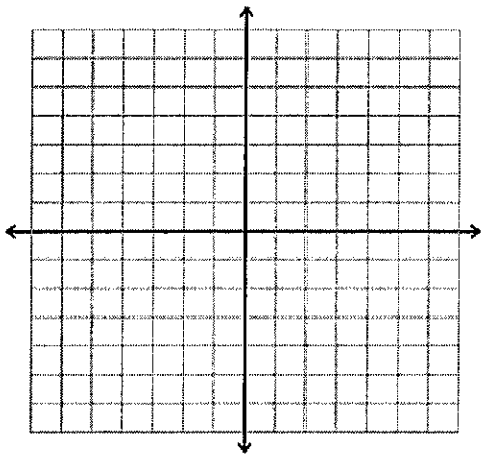
$$R_{180°}(x, y) = (-x, -y)$$

$$R_{270°}(x, y) = (y, -x)$$

Example 1

Plot the point A (3, 5)

- $R_{90}(A)$
- $R_{180}(A)$
- $R_{270}(A)$

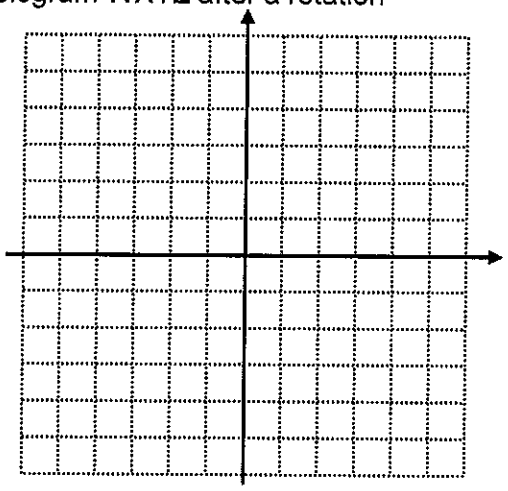


Example 2:

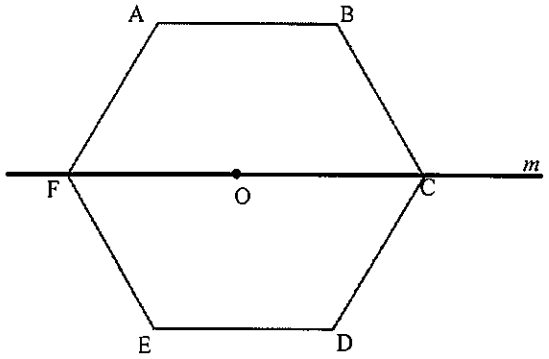
Graph the vertices of parallelogram WXYZ.

W(-3,5)
X(1,5)
Y(2,3)
Z(-2,3)

Graph and state the coordinates of parallelogram WXYZ after a rotation of 90 degrees with respect to the origin.



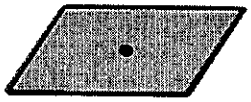
Example 3:



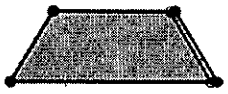
1. $R_{O,60}$ (C) = ( )
2. $R_{O,120}$ (A) = ( )
3. $R_{O,300}$ (E) = ( )
4. $R_{O,180}$ (D) = ( )
5. $R_{O,-120}$ (C) = ( )
6. $R_{O,-60}$ (B) = ( )
7. r_m (E) = ( )
8. r_m (C) = ( )

Rotational Symmetry

Definition: A geometric figure has rotational symmetry if the figure appears unchanged after a rotation about a point by an angle whose measure is strictly between 0° and 360°. The angles of 0° and 360° are excluded since they represent the original position (nothing new happens). The angles of rotational symmetry will be factors of 360.

Example #1	Example #2	Example #3	Example #4
			

Here are a few non-examples of shapes that DO NOT have rotation symmetry.

NON - Example #1	NON - Example #2	NON - Example #3	NON - Example #4
 (the colors don't work)	 (numbers don't work)		

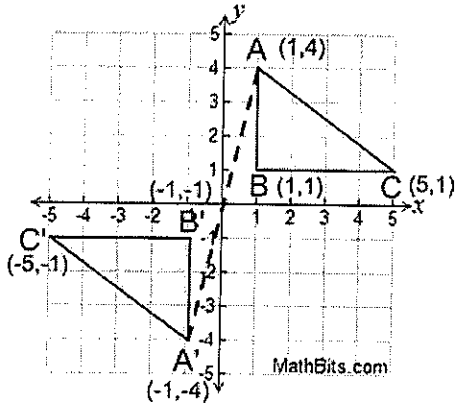
Regular POLYGONS have rotational symmetry.

equilateral triangle | square | regular pentagon | regular hexagon | regular heptagon | regular octagon

Point Reflections

Equivalent to a rotation of 180 about point.

- Reflect in origin (0,0): Equivalent to $R_{O,\,180}$

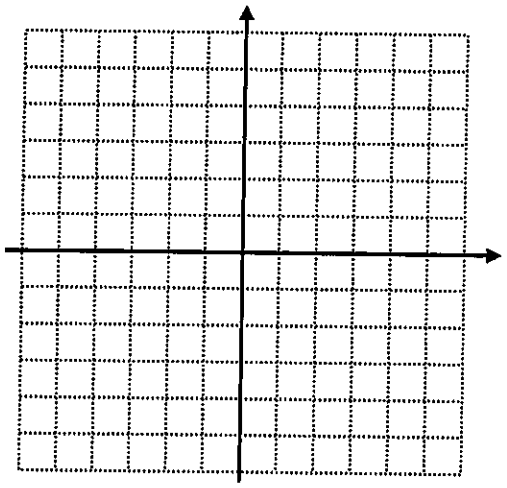


In a point reflection in the origin, the image of the point (x,y) is the point $(-x,-y)$.

Example 4

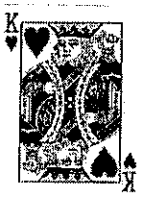
Plot point A (-2, 4)

- $r_{(0,0)}A$
- $r_{(1,\,1)}A$
- $r_{(-1,\,-1)}A$



Point Symmetry

A figure has point symmetry if it has 180 rotational symmetry. (ORDER 2)



MOM

Name _____ Date _____

Geometry CC: Rotations HW

1 If the letter **P** is rotated 180 degrees, which is the resulting figure?

1) **d**
2) **a**
3) **q**
4) **b**

2 What are the coordinates of A', the image of $A(-3,4)$, after a rotation of 180° about the origin?

1) (4,−3)
2) (−4,−3)
3) (3,4)
4) (3,−4)

3 What are the coordinates of M', the image of $M(2,4)$, after a counterclockwise rotation of 90° about the origin?

1) (−2,4)
2) (−2,−4)
3) (−4,2)
4) (−4,−2)

4 What is the image of point (8,−4) under the rotation $R_{90°}$ about the origin?

1) (8,4)
2) (4,8)
3) (−4,8)
4) (−4,−8)

6 The point (−2,1) is rotated -270° about the origin. What are the coordinates of its image?

7 What is the image of $R_{O,90}(1, 2)$?

8 Write the coordinates of P', the image of $P(5,-1)$ after a reflection through the point (2, -3).

9 What is the image of (5,1) under a counterclockwise rotation of -90°?

10 If point $P(3,-2)$ is reflected through the point (1, 1), what is the image of P'? [GRAPH IT]

11 The coordinates of the endpoints of $\overline{BC}$ are $B(5,1)$ and $C(-3,-2)$. Under the transformation $R_{O,90}$, the image of $\overline{BC}$ is $\overline{B'C'}$. State the coordinates of points B' and C'.

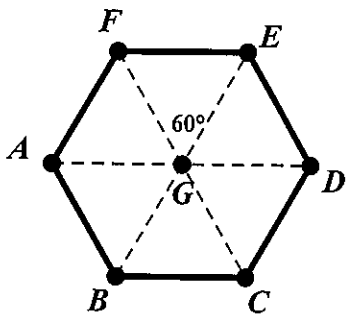
2. Determine the name of the point that meets the given conditions.

a) $R_{G,60°}(A) =$ _____ b) $R_{G,180°}(B) =$ _____

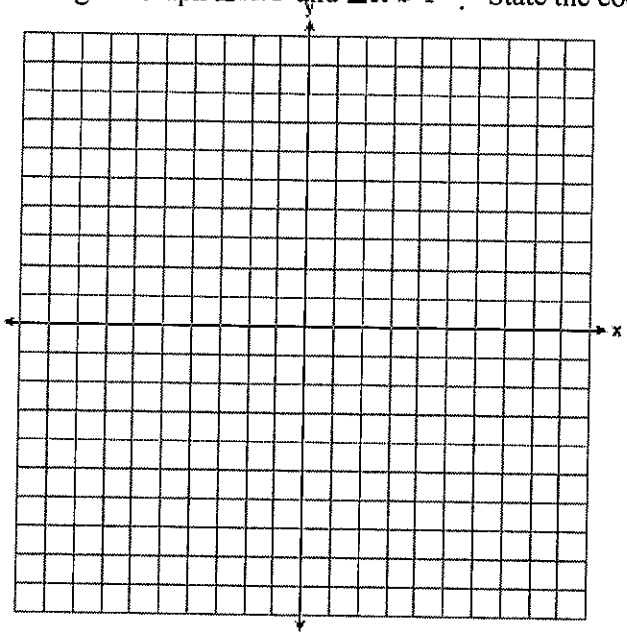
c) $R_{G,300°}(D) =$ _____ d) $R_{G,-120°}($_____$) =$ B

e) $R_{G,240°}(E) =$ _____ f) $R_{G,-240°}(F) =$ _____

g) $R_{A,60°}(B) =$ _____ h) $R_{C,120°}(D) =$ _____



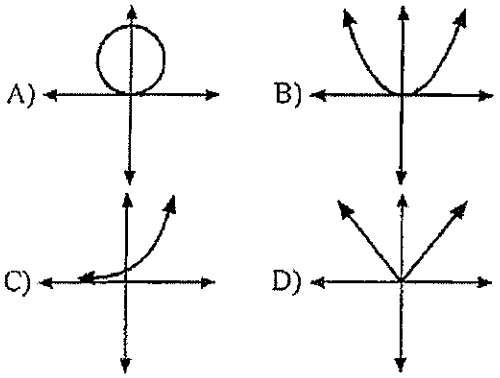
13. The coordinates of the vertices of ΔRST are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Triangle $R'S'T'$ is the image of ΔRST after a rotation of 90° about the origin. Graph ΔRST and $\Delta R'S'T'$. State the coordinates of the vertices of $\Delta R'S'T'$.



14. Which letter has point symmetry?

A) E B) H C) C D) T

15. Which graph has point symmetry?



16. Which figure does not have rotational symmetry?

A) regular hexagon B) equilateral triangle
C) trapezoid D) rectangle

17. Which polygon has rotational symmetry of 90°?

A) square B) regular pentagon
C) regular hexagon D) equilateral triangle

18. Which figure has 120° rotational symmetry?

A) rhombus B) regular pentagon
C) square D) equilateral triangle

19. Which figure has 60° rotational symmetry?

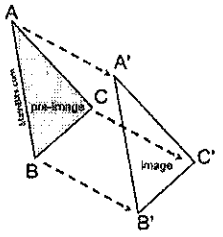
A) regular octagon B) regular hexagon
C) equilateral triangle D) square

20. Which geometric figure has 72° rotational symmetry?

A) regular pentagon B) rhombus
C) square D) equilateral triangle

Unit 2 Lesson 4: Translations

Definition: A translation is a rigid transformation of the plane that moves every point of a pre-image a constant distance in a specified direction.



Properties preserved under a **translation** from the pre-image to the image.

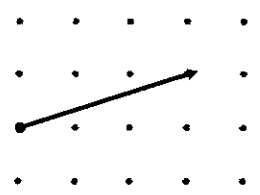
1. **distance** (lengths of segments remain the same)
2. **angle measures** (remain the same)
3. **parallelism** (parallel lines remain parallel)
4. **collinearity** (points remain on the same lines)
5. **orientation** (lettering order remains the same)

http://www.youtube.com/watch?v=A05n32Bl0aY

Relationship of Pre-image to Image	Translation
Distance of the paths	Same (congruent)
Relationship of the paths	parallel
Orientation	Same
Special Points	None all points in the plane move

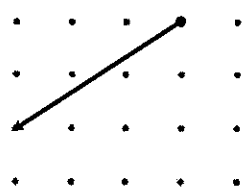
Examples

Given the following diagrams describe the translation using both notations

1. 

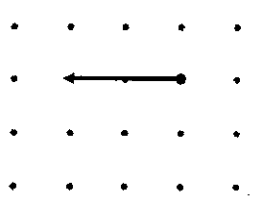
Mapping:

Vector Notation:

2. 

Mapping:

Vector Notation:

3. 

Mapping:

Vector Notation:

Example 4:

Determine the translation rule from the pre-image and image.

a) A (1,2) A' (-6,5) T (x,y) ---------- > (_____ , _____)

b) B (-2,10) B' (1,5) T (x,y) ---------- > (_____ , _____)

c) If point C(2, 3) is mapped onto C'(-5, 9) under a certain translation, what are the coordinates of D', the image of D(-1, -4) after the same translation.

Example 5:

Given a translation rule, determine the missing point.

a) T (x,y) -------> (x - 5, y + 3) A (-3,2) ⟶ A' (_____ , _____)

b) T (x,y) -------> (x + 2, y – 3) B (6,2) ⟶ B' (_____ , _____)

Example 6:

Given a translation rule, determine the missing point.

a) T (x,y) -------> (x - 2, y + 9) A(_____ , _____) ⟶ A' (-3,2)

b) T (x,y) -------> (x , y – 4) B' (_____ , _____) ⟶ B' (-3 , 6)

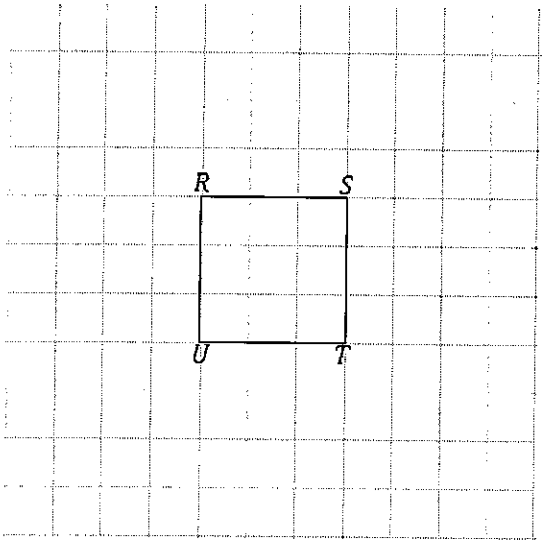
Example 7:

Under a certain translation A maps onto A'.
Using the translation rule, complete the chart.

A(2, 5)	A'(-2, 9)
B( )	B'(0, 6)
C(-1, 3)	C'( )

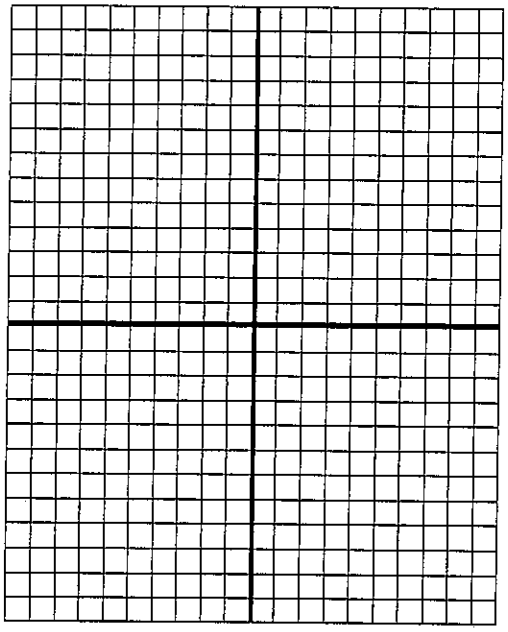
Example 8:

Translate the figure $T_{<-3, -2>}$
Draw the vector that defines the translation. Then perform the translation.



Example 9:

The vertices of ΔRST are R(-6, 5), S(-7, -2) and T(1, 4). Graph, state and label the coordinates of ΔR'S'T', the image of ΔRST after the transformation $T_{<-2, 3>}$. Is this transformation a rigid motion? Explain.

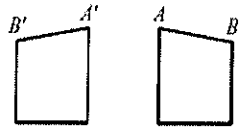


Unit 1-2 Rigid Motions REVIEW
Geometry CC

1. Which of the following capital letters has at least one line of symmetry?

(1) F (2) R (3) H (4) L

2. Do the figures below have the same or opposite orientation?



3. What is the rotation symmetry order of a regular hexagon?

(1) 1 (2) 6 (3) 60 (4) 180

4. Which of the following is not a rigid motion?

(1) Reflection (2) Rotation
(3) Stretch (4) Translation

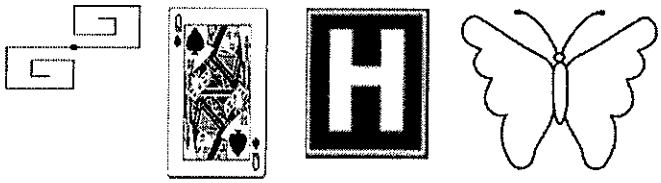
5. Which term describes a transformation that does not alter a figures shape or size?

(1) Symmetry (2) Similarity
(3) Isometry (4) Transformation

6. If you have rotational symmetry of 90 degrees, then your order is

(1) 2 (2) 4
(3) 6 (4) 8

7. Which shape has doesn't have point symmetry?



8. Line symmetry is the same as:

(1) Rotational Symmetry
(2) Point Symmetry
(3) Translation Symmetry
(4) Reflectional Symmetry

9. Which transformation changes the orientation of the shape?

(1) Rotation (2) Dilation
(3) Translation (4) Line Reflection

10. $\Delta A'B'C'$ is the image of ΔABC under transformation G. Line m is the perpendicular bisector of $\overline{AA'}, \overline{BB'}, and\ \overline{CC'}$. Which describes the transformation G?

(1) A rotation (2) A line reflection
(3) A translation (4) A dilation

11. After a figure is rotated, A = A'. Which statement(s) are true?

(1) The center of rotation is A
(2) A is on the line of reflection
(3) Both 1 and 2
(4) Neither 1 or 2

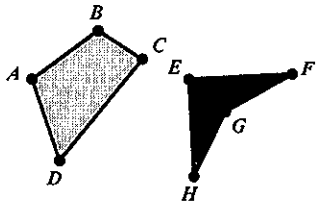
12. A figure is transformed in a plane such that no point maps to itself. Which transformation must it be?

(1) Line Reflection (2) Translation
(3) Rotation (4) Dilation

13. Given the coordinate rule $G(x, y) \rightarrow (x - 8, 2y)$ determine the image of A (- 3, 5).

14. Using the same coordinate rule from **question13**, determine the pre-image if B'(5, 12).

15. Tyler is given a question on a test about transformations. He is given the example below and asked about the relationship between the pre-image and image. He says that the picture is an example of a rigid motion. **Is he correct? Explain why you agree or disagree.**



16. What are the coordinates of A', the image of $A(-3,4)$, after a rotation of 180° about the origin?

(1) (4, -3) (3) (-4, -3)
(2) (3, 4) (4) (3, -4)

17. What is the image of $A(5,2)$ under $R_{O,90}$?

(1) (-5, 2) (3) (5, -2)
(2) (2, 5) (4) (-2, 5)

18. If translation T maps point A(−3, 1) onto point $A'(5, 5)$, which is translation T?

(1) $T_{2,4}$ (3) $T_{2,6}$
(2) $T_{8,6}$ (4) $T_{8,4}$

19. Point A is located at $(4,-7)$. The point is reflected in the x-axis. Its image is located at

(1) (-4, 7) (3) (-4, -7)
(2) (4, 7) (4) (7, -4)

20. Reflecting (5, 1) in the y-axis yields an image of

(1) (5, -1) (3) (-5,-1)
(2) (5, 1) (4) (-5, 1)

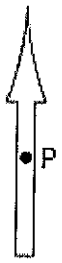
21. What is the image of the point (−5,2) under the translation $T_{3,-4}$?

(1) (-9, 5) (3) (-8, 6)
(2) (-2, -2) (4) (-15, -8)

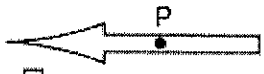
22. A translation moves $P(3,5)$ to $P'(6,1)$. What are the coordinates of the image of point (−3,−5) under the same translation?

(1) (0, -9) (3) (-5, -3)
(2) (-6, -1) (4) (-6, -9)

23. The accompanying diagram shows the starting position of the spinner on a board game.

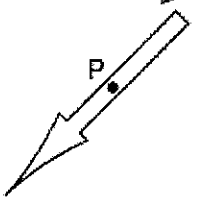


How does this spinner appear after a 270° counterclockwise rotation about point P?

1)


2)


3)


4)


24. The image of point (−2,3) under translation T is (3,−1). What is the image of point (4,2) under the same translation?

(1) (-1, 6 (3) (0, 7)
(2) (5, 4) (4) (9, -2)

25. What is the image of (5,−2) under the transformation $r_{y=x}$?

(1) (-5, 2) (3) (5, 2)
(2) (2, 5) (4) (-2, 5)

26. What is the image of point (−3,−1) under a reflection in the point (1, 3)? GRAPH IT

27. When point A(-2, 5) is reflected in the line y = 1, the image is

(1) (-2, -3) (3) (0, 5)
(2) (5, 2) (4) (4, 5)

28. Which transformation in *not* an isometry?

(1) translation (3) rotation
(2) dilation (4) line reflection

29. Which transformation does *not* preserve orientation?

(1) translation (3) rotation
(2) dilation (4) line reflection

30. Which transformation does *not* preserve distance?

(1) translation (3) rotation
(2) dilation (4) reflection

31. Which letter has point symmetry?

(1) B (2) N (3) T (4) A

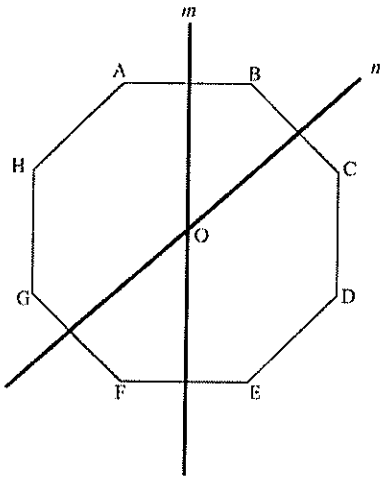
32. Which combination of letters has both vertical and horizontal reflectional (line) symmetry?

(1) MOM (3) XOOX
(2) ZOO (4) NOON

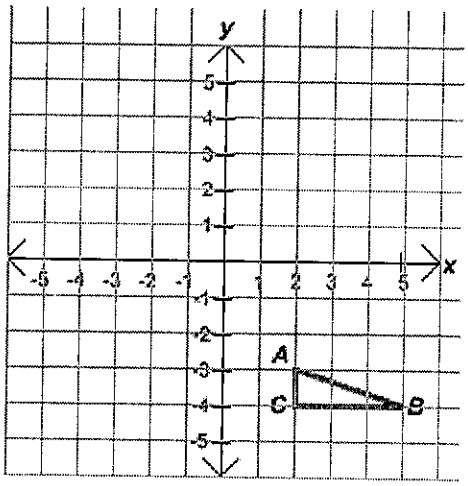
33. State 2 equivalent rotations
(1 positive, 1 negative)

34. Using the diagram below determine:
(not multiple choice)

a. $R_{45}(D)$
b. r_O (A)
c. r_m (H)
d. r_n ($\overline{AB}$)
e. $R_{-135}(B)$

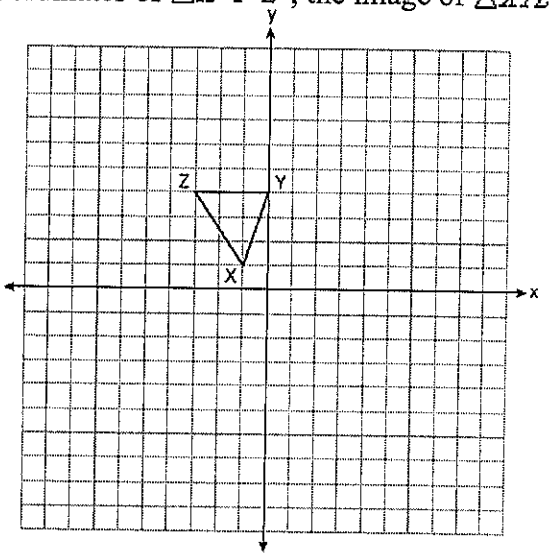


35. On the set of axes below, graph
Draw the image produced when
△ABC is R_{270}.
State the new coordinates.



36. On **GRAPH PAPER** graph and label triangle TAP with coordinates $T(-1,4)$, $A(2,4)$, and $P(2,0)$. Graph and label $\Delta T'A'P'$, the image of ΔTAP after the translation $(x,y) \rightarrow (x-5, y-1)$.

37. Triangle XYZ, shown in the diagram on the right, is reflected over the line $x = 2$. State the coordinates of $\Delta X'Y'Z'$, the image of ΔXYZ.



38. Under a translation, all pre-image to image paths are PARALLEL. (True or False)

39. Under a reflection, all pre-image to image paths are congruent. (True or False)