Advanced Mathematics Final Exam Topics

- **Factoring**
  - * Look for GCF first
    $6ab^2 - 12b \rightarrow 6b(ab-2)$
  - * AM method
    $x^2 - \overset{A}{x} - \overset{M}{42} \rightarrow (x-7)(x+6)$
  - * DOTS (Difference of Two Squares)
    $x^2 - 4 \rightarrow (x+2)(x-2)$
  - * Slip & Slide (Don't forget to divide!)
    $3x^2 - 5x + 2$
    $x^2 - 5x + 6 \rightarrow (x-\frac{3}{3})(x-\frac{2}{3}) \rightarrow (x-1)(3x-2)$
  - *Grouping

$2a^3 - a^2b + 10a - 5b$	1) split
$a^2(2a-b) + 5(2a-b)$	2) GCF (make sure ( ) match)
$(a^2+5)(2a-b)$	3) rewrite

  - *If asked to solve quadratic, use T-chart:
    $x^2 - x - 42 \rightarrow (x-7)(x+6)$

$x-7=0$	$x+6=0$
$\boxed{x=7}$	$\boxed{x=-6}$

- **Exponent Rules**
  - * product rule (add) $(x^8)(x^4) = x^{12}$
  - * power rule (multiply) $(x^3)^2 = x^6$
  - * quotient rule (subtract) $\frac{x^4}{x^2} = x^2$ or $\frac{x^2}{x^4} = \frac{1}{x^2}$
  - * negative exp. rule $x^{-2} = \frac{1}{x^2}$ or $\frac{1}{x^{-3}} = x^3$
  - * zero exp. rule $x^0 = 1$

- **Fractional Exponents ↔ Radical**
  - * $x^{\frac{1}{2}} = \sqrt{x}$
  - * $yx^{\frac{2}{3}} = y\sqrt[3]{x^2}$
  - * $x^{\frac{a\ (\text{inside})}{b\ (\text{outside})}}$

- Solve Exponential Equations

  * $3^{2x} = 27^{x+3}$

    $3^{2x} = (3^3)^{x+3}$ — 1) make base the same

    $2x = 3(x+3)$ — 2) Distribute & set equal

    $2x = 3x + 9$ — 3) solve

    $-3x \quad -3x$

    $-x = 9 \rightarrow x = -9$

- Radicals

  * simplify

    $\sqrt{27} \rightarrow \sqrt{9}\sqrt{3} \rightarrow 3\sqrt{3}$

    $\sqrt{24a^3} \rightarrow \sqrt{4a^2}\sqrt{6a} \rightarrow 2a\sqrt{6a}$

    $\sqrt[3]{40x^5y^6} \rightarrow \sqrt[3]{8x^3y^6}\sqrt[3]{5x^2} \rightarrow 2xy^2\sqrt[3]{5x^2}$

  * Solve $10 + \sqrt{10x - 1} = 13$

    $(\sqrt{10x-1})^2 = (3)^2$ — 1) isolate radical

    $10x - 1 = 9$ — 2) square both sides

    $10x = 10$ — 3) solve

    $x = 1$

- Imaginary $(i = \sqrt{-1})$

  *Simplify (I won, I won, neg. in the middle)

  Divide by 4 - $i \rightarrow .25$, $-1 \rightarrow .5$, $-i \rightarrow .75$, $1 \rightarrow$ whole #

  $i^{33} = i$ $(33 \div 4 = 8.\underline{25})$

  $i^{28} = 1$ $(28 \div 4 = \underline{7})$

  * multiply $(i^2 = -1)$ $(3+5i)(2-i)$

    $6 - 3i + 10i - 5i^2 \rightarrow 6 + 7i - 5(-1) = 11 + 7i$

* Radicals & imaginary #'s

$3\sqrt{-4} + 7\sqrt{-8}$

$3 \cdot 2i + 7 \cdot \sqrt{-4}\sqrt{2}$

$6i + 7 \cdot 2i\sqrt{2}$

$6i + 14i\sqrt{2}$

$\sqrt{-27} - \sqrt{-12}$

$\sqrt{-9}\sqrt{3} \quad \sqrt{-4}\sqrt{3}$

$3i\sqrt{3} - 2i\sqrt{3}$

$i\sqrt{3}$

- Solve for variable

* solve a: $g = ca - b$

$+b \quad +b$

$\frac{g+b}{c} = \frac{ca}{c} \rightarrow \frac{g+b}{c} = a$

- Inverse $f(x) = 4x - 3$

$y = 4x - 3$ 1) change $f(x)$ to y

$x = 4y - 3$ 2) switch x & y

$+3 \quad +3$ 3) solve for y

$\frac{x+3}{4} = \frac{4y}{4} \rightarrow f^{-1}(x) = \frac{x+3}{4}$

- End behavior

1st #	highest exp. even	highest exp. odd
positive	↑ ↑	↓ ↑
negative	↓ ↓	↑ ↓

$f(x) = -3x^4 + 2x^3 - 2 - 5x$

neg & even ↓ ↓

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to -\infty$

- Shifting Functions

$g(x) = f(x \pm a) \pm b$

$(+$ left, $-$ right$)$

$+$ up, $-$ down

- Growth/decay $y = a(1 \pm r)^t$

5% - (1+.05) 7% - (1-.07)
1.05 .93

a - initial amount
r - growth rate (as decimal)
decay rate (as decimal)
t - time

- Add/subtract matrices (combine like elements)

$$\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} -5 & 6 \\ 7 & -8 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix}$$

- Long Division (Synthetic Division)

*Divide $9x^5 - 9x^4 - x^3 - 12x^2 + x - 11$ by $3x - 5$

$3x - 5 = 0$ 1) solve for x
$+5 \quad +5$ 2) Division →
$\frac{3x}{3} = \frac{5}{3}$ 3) keep 1st #
$x = \frac{5}{3}$ 4) mult. & combine until you reach end

$\frac{5}{3}$ | 9 -9 -1 -12 1 -11
↓ 15 10 15 5 10
9 6 9 3 6 -1
↑ remainder

5) rewrite answer w/ remainder as fraction: $9x^4 + 6x^3 + 9x^2 + 3x + 6 - \frac{1}{3x-5}$

- System of Equations (Linear/Quadratic)

* $y = x^2 - 6x + 9$
$y + x = 5 \longrightarrow y = -x + 5$

1) set equal & solve
$x^2 - 6x + 9 = -x + 5$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$
$x = 4 \mid x = 1$

2) plug in x's to find y
$y = -(4) + 5$ $y = -(1) + 5$
$y = 1$ $y = 4$

$(4, 1)$ & $(1, 4)$

- Rational Expressions (Common Denominator)
  * Add/subtract

Common denom: $4(x-1)$: $\frac{6}{x-1} - \frac{5x}{4} \rightarrow \frac{6(4)}{4(x-1)} - \frac{5x(x-1)}{4(x-1)} = \frac{24-(5x^2-5x)}{4(x-1)}$

$$= \frac{-5x^2+5x+24}{4(x-1)}$$

* Multiply (Factor & cross out)

$$\frac{x^2+5x}{x^2-25} \cdot \frac{x^2-x-20}{2x^3} \rightarrow \frac{x(x+5)}{(x+5)(x-5)} \cdot \frac{(x-5)(x+4)}{2x^3} = \frac{x+4}{2x^2}$$

*Divide (keep, change, flip)

$$\frac{x^2-4x-21}{x^2-49} \div \frac{3x+9}{x^2+7x} \rightarrow \frac{x^2-4x-21}{x^2-49} \cdot \frac{x^2+7x}{3x+9}$$

$$\rightarrow \frac{(x-7)(x+3)}{(x-7)(x+7)} \cdot \frac{x(x+7)}{3(x+3)} = \frac{x}{3}$$

- Logic: Contrapositives are logically equivalent (negate & switch)

*"If I am tired, then I ran today." $(p \rightarrow q)$
"If I did not run today, then I am not tired." $(\sim q \rightarrow \sim p)$

- Trig

S	A
T	C

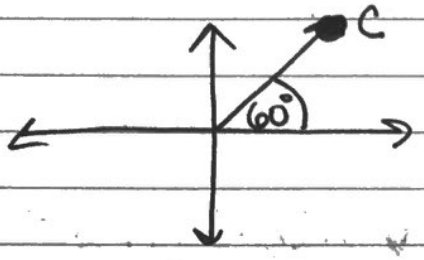
*where positive/negative

"All students take calc"

* Radian ↔ Degrees

$\frac{36^\circ}{180} \rightarrow \frac{1}{5} \rightarrow \frac{\pi}{5}$ $\qquad$ $\frac{2\pi}{3} \rightarrow \frac{2(180)}{3} = 120^\circ$

* points on unit circle $(\cos\theta, \sin\theta)$

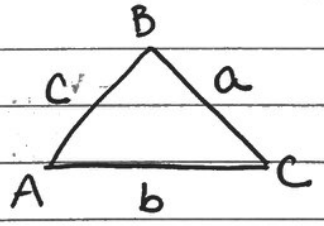


Coordinates of C:
$(\cos 60, \sin 60)$
$C\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

*Know hand rules

*Law of sines/cosines

$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\qquad$ $a^2 = b^2 + c^2 - 2(b)(c)\cos A$



* largest ∠ opp largest side

*smallest ∠ opp smallest side

*Area of a Δ $\qquad A = \frac{1}{2}ab\sin C$
(2 sides + the ∠ between them)

- Quadratic Formula (use when can't factor) - given on exam

• Cramer's Rule (Matrices & systems)

$-4x+9y=9$
$x-3y=-6$

1) write as matrix $\begin{matrix} x & y & = \end{matrix}$
$\left[\begin{array}{cc|c} -4 & 9 & 9 \\ 1 & -3 & -6 \end{array}\right]$

2) Find determinant $\begin{bmatrix} -4 & 9 \\ 1 & -3 \end{bmatrix} = (-4)(-3)-(1)(9)$
$= 12-9 = 3$

3) Find x & y (both ÷ by determinant)

$\begin{matrix} = & y \end{matrix}$
$x = \dfrac{\begin{bmatrix} 9 & 9 \\ -6 & -3 \end{bmatrix}}{3} = \dfrac{9(-3)-(9)(-6)}{3} = \dfrac{27}{3} = 9$

$\begin{matrix} x & = \end{matrix}$
$y = \dfrac{\begin{bmatrix} -4 & 9 \\ 1 & -6 \end{bmatrix}}{3} = \dfrac{(-4)(-6)-(1)(9)}{3} = \dfrac{15}{3} = 5$

4) Answer: $(9, 5)$

Use for Notes: